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# Search, Heterogeneity, and Optimal Income Taxation

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November 9, 2009

**Abstract**

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D [illegible]. N [illegible], M [illegible] (1982), H [illegible] (1990) [illegible] A [illegible] (1999) [illegible]), [illegible]. H [illegible], [illegible] (2001) [illegible]. I [illegible].

[illegible]. I [illegible]. I [illegible] / [illegible] / [illegible]

## 2 Model

$(H)$ $(L)$ $l_k$, $k = H; L$, $q_m$, $m = H; L$. $y_{km} > 0$, $y_{Hm} > y_{Lm}$.

$_k \in 0; 1$, $c_w(\ _k)$, $c_w(0) = 0$, $c'_w(0) = 0$, $\delta_k \to$ $c'_w(\ _k) = +\infty$. $v_m$, $c_\pi(v_m)$

[illegible]

## 2.1 The matching technology

[illegible]

, 1 – ff

. A M P (1999) ,

. H , P (1996) B D -

## 2.2 Output sharing

[illegible]

## 2.4 Private expected utility functions

fi , , $v_H q_H = \quad _m q_m$.

$k$

$$U_k = -c_w(\ _k) + \ _k \ M(\ ) \ \frac{v_H q_H}{_m v_m q_m} \ _{kH} y_{kH} + \frac{v_L q_L}{_m v_m q_m} \ _{kL} y_{kL} \ + (1 - M(\ ))0 \ + (1 - \ _k)0$$

$$U_k = -c_w(\ _k) + \ _k M(\ ) E_{(m)} \ _{km} y_{km}; \tag{5}$$

$E_{(m)}$ . A , , $c(\ )$ , . O , $M(\ )$, . I , $1 - M(\ )$, , , . I , (

[illegible]

# 3 Optimal search intensity and market ine ciencies

[illegible]

## 3.1 Social Optimum

A [illegible]:

$$W = {}_{\delta,v} \quad {}_{k} l_k U^k + {}_{m} q_m V^m$$

$$. \quad . \quad {}_{k} \geq 0; \quad v_m \geq 0:$$

[illegible] (1), (2), (5), [illegible] (6), [illegible] -91.21 [illegible]

$$E_{(m)}y_{km} - (1 - \ )E_{(k)}E_{(m)}y_{km} = \ E_{(k)}E_{(m)}y_{km} + E_{(m)}y_{km} - E_{(k)}E_{(m)}y_{km}.$$

A ... ( ... ) ... ( $E_{(k)}E_{(m)}y_{km}$ ), ... ff ... ( ... ( ... ) ... ) ... , $E_{(m)}y_{km}$, ... , $E_{(k)}E_{(m)}y_{km}$. ... ( ... ) ff ... .

... :

... 1 . I ...

$$c'_\pi(\bar{v}_H) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kH} - \ E_{(k)}E_{(m)}y_{km}$$
$$c'_\pi(\bar{v}_L) = \frac{M(\bar{\ })}{\bar{\ }} E_{(k)}y_{kL} - \ E_{(k)}E_{(m)}y_{km} \quad \Bigg| \quad \bar{\ } \lesseqgtr 1; \ \bar{v}_H > 0; \ \bar{v}_L > 0 \quad ; \qquad (13)$$

$$c'_\pi(\bar{v}_H) = \frac{M(\ )}{} E_{(k)}y_{kH} - \ E_{(k)}E_{(m)}y_{km}$$
$$c'_\pi(0) \geq \frac{M(\ )}{} E_{(k)}y_{kL} - \ E_{(k)}E_{(m)}y_{km} \quad \Bigg| \quad \bar{\ } \lesseqgtr 1; \ \bar{v}_H > 0; \ \bar{v}_L = 0 \quad . \qquad (14)$$

## 3.2 Decentralized equilibrium

A ... $k$ ...

$$\max_{\delta_k} \ U_k = -c_w(\ _k) + \ _k M(\ )E_{(m)} \ _{km}y_{km}$$
$$\text{s.t.} \ _k \geq 0; \qquad (15)$$

... $C_k = c_w(\ _k)$ ... , ... $B_k = \ _k M(\ )E_{(m)} \ _{km}y_{km}$ ... fi ... .

K ${}^{2}$

$$-c_w'(\ {}_k) + M(\ )E_{(m)}\ {}_{km}y_{km} \le 0$$

$${}_k \ge 0 \qquad (16)$$

$$(-c_w'(\ {}_k) + M(\ )E_{(m)}\ {}_{km}y_{km})\ {}_k = 0;$$

$_k$ . . . $M(\ )$, . . . F . . . ff . . . ff . . .

I . . . $_{LL} \neq 0$ ( $_{LH} = 0$), . . . $y_{km} > 0$,

fi (06 )2 158.730 ( ) -395. 4.9812163(0),)-373( )-333( ( )-350( 5. 4

. D l ll K (1992), ll B B (2002),

ff ll ff ;

fi fi

. O fi

**4.** *I*

$$c'_w(\ \ _k) = M(\ \ )(1 - \ \ ^w_k)w_k \qquad \lesseqgtr 1; \quad ; \qquad (22)$$

$$c'_\pi(v_m) = \frac{M(\ \ )}{}(1 - \ \ ^\pi_m)\ \ _m \qquad _k > 0;\ v_m > 0$$

$$c'_w(0) \geq M(\ \ )(1 - \ \ ^w_L)w_L \qquad \lesseqgtr 1; \quad : \qquad (23)$$

$$c'_\pi(0) \geq \frac{M(\ \ )}{}(1 - \ \ ^\pi_L)\ \ _L \qquad _L = 0;\ v_L = 0$$

## 4.1 Characterizing externalities through Pigou taxes

, P $(\check{\ }^w_H; \check{\ }^w_L)$, P $(\check{\ }^\pi_H; \check{\ }^\pi_L)$. L ( ) $\check{R}$, P , , $LS$:

$$\check{R} = (\ \ _k\ \ l)M(\ \ )\ \ \frac{_H l_H}{_k\ \ l}\check{\ }^w_H w_H + \frac{_L l_L}{_k\ \ l}\check{\ }^w_L w_L + \frac{v_H q_H}{_m vq}\check{\ }^\pi_H\ \ _H + \frac{v_L q_L}{_m vq}\check{\ }^\pi_L\ \ _L$$

$$0 = \check{R} - \ \ _k l_k + \ \ _m l_m\ \ LS;$$

$(\ \ _k\ \ l)M(\ \ ) = N$ .

$k$

$$U_k = -c_w\ \ \frac{z^w_k}{M(\ \ )\,w_k}\ \ + LS + (1 - \check{\ }^w_k)z^w_k \qquad (24)$$

, $LS$, ff

. fi ,

$c'_w$

[illegible] ( [illegible] P [illegible] 5)
[illegible]. CR [illegible], [illegible]
[illegible]; [illegible]
[illegible], [illegible].
[illegible], P [illegible]
[illegible], [illegible]
[illegible].
[illegible], [illegible]
[illegible] ff [illegible]. I [illegible]
[illegible] fi [illegible], [illegible] fi [illegible], L [illegible],
[illegible].

N [illegible], [illegible]
[illegible] ff [illegible] $(g = 0)$. [illegible]
[illegible], [illegible]
[illegible]. I [illegible]

$y_{LL} > 0$,

## 4.2 Optimal income taxes with positive government revenue

$R$,

$$W = \sum_k l_k U^k + \sum_m q_m V^m;$$

(1), (2), (5), (6), $w_k = E_{(m)} \; {}_{km} y_{km}$, $z^w_k = {}_k M(\;) w_k$, ${}_m = E_{(k)}(1 - {}_{km}) y_{km}$, $z^\pi_m = v_m \frac{M\,\theta}{\theta} \; {}_m$ 4,

$$W = \sum_k l_k \left( -c_w \frac{z^w_k}{M(\;)\, w_k} \right) + \sum_m q_m \left( -c_\pi \frac{z^\pi_m}{\frac{M\,\theta}{\theta} \; {}_m} \right) + (\textstyle\sum_k l)M(\;)\, E_{(k)} E_{(m)} y_{km};$$

$(\sum_k l)M(\;) = N$

R,

$$R \le (\textstyle\sum_k l)M(\;) \left[ \frac{{}_H l_H}{\sum_k l} \, {}^w_H w_H + \frac{{}_L l_L}{\sum_k l} \, {}^w_L w_L + \frac{v_H q_H}{\sum_m vq} \, {}^\pi_H \; {}_H + \frac{v_L q_L}{\sum_m vq} \, {}^\pi_L \; {}_L \right]; \qquad (30)$$

$(\sum_k l)M(\;) = M$ (30),

[17]

$l_{dld\ dldslsdzdHdzdHdzddkqwmldMdHddkdddHdddLddLdddkqwmldMdLddkisiddLqwdddHdzdHdzddkmqwdldMdHdddldkdldddldHddqdddLddLdddkmqwdldMdLdd}$

$$W = \sum_k l_k \left( - c_w \frac{z_k^w}{M(\ )\, w_k} \right) + \sum_m q_m \left( - c_\pi \frac{z_m^\pi}{\frac{M\ \theta}{\theta}\ m} \right)$$
$$+ (\sum_k l) M(\ ) \left( \frac{{}_H l_H}{\sum_k l}(1 - {}^w_H) w_H + \frac{{}_L l_L}{\sum_k l}(1 - {}^w_L) w_L \right)$$
$$+ \frac{v_H q_H}{\sum_m vq}(1 - {}^\pi_H)\ {}_H + \frac{v_L q_L}{\sum_m vq}(1 - {}^\pi_L)\ {}_L + R.$$

[illegible] (22) ( C (1978), D (1998) [illegible]

[illegible] (22) [illegible]

$$\max_{\tau_k^w, \tau_m^\pi} W = \sum_k l_k \left( - c_w \frac{z_k^w}{M(\ )\, w_k} \right) + \sum_m q_m \left( - c_\pi \frac{z_m^\pi}{M\ \theta} \right)$$

[illegible] P [illegible] 8 [illegible] A [illegible]. [illegible]

[illegible] $=0$ [illegible] $=0$. [illegible] (33). [illegible] H [illegible] P [illegible] 5 [illegible] A [illegible] A). H [illegible]

[illegible] $= \infty$, [illegible] $= \infty$, [illegible]

**4.2.2** [illegible]

[illegible] 1 [illegible]

/

( ),

( D (1998)). fi

’ (P 11 ),

, .

P 11 ( ) 

. A

P 5,

**12.**

$$i)\ @\ \frac{w_H}{w_L} = @\ \frac{E_{(m)}\ Hm}{E_{(m)}\ Lm} < 0 \quad (36)$$

$$ii)\ @\ \frac{w}{\pi} = @(\ ) < 0 \quad (37)$$

P 11 A . B

( P 5).

( ),

P 12

B B (2002)

## 5 Conclusion

I

(1 ) "H Effi F ," I E , 40, 827-849.

(2002) "O L ," J E , 85, 53-97.

(1 4) "Eff B fi F ," P .

(1 ) "M Cl ," J E , 112, 141-168.

(1 ) "L - P F : M E ," E J , 109, 307-334.

(1 ) "O R : M R ," A E , 68, 756-768.

(1 1) "M C , F Effi ," J E , .89, 798-812.

(1 2) "A D M E ," J E , .90, 881-894.

(1 2) " D Effi E ," E , .49, 217-227.

(1 ) "O I : A E P O M R ," A E , .88, 83-95.

(1 ) " P ," E J , 7, 46-70.

(1 3) "O O P I ," J E , 2, 357-376.

(1 ) "H D P Aff E ? C O E ," M P .

, . (1 0) "Effi I ," *J E* , 51, 8999.

., . (1 5) "P , , ," P , D E , .

, . . (1 0) "O Effi M R M ," *E* , 57, 279-298.

, . (1 ) " , F , ," *E* , 32, D E , .

, ., . (1 ) " P G E P M B ," *L E* , 3, 6580.

., . (1 ) "L R , G J B J ," *E* , *C* , P .

, ., . (1 3) "

., (1 ) "L M D Eff  
," J E , 33, 457495.

(2000) "A M ," E , 68, 343-369.

(2001) "M , , H " A  
, 1(2001), I . 1, A . 5.

. (1 ) "P F E P ?,"  
E , 25, 223264.

. (1 ) "O P F L M ," L  
E , 6, 435452.

. (1 ) "P Effi O N N  
E ," I : H E , .2.

. (1 ) " - P Effi ," J  
E , 6, 239262.

. (1 ) " , P P D .,"  
I F , 6, 239262.

, . (1 ) " P F E ," I : ,  
.B. (E .), F C ., M M , L .

# Appendices:

## A Proofs of the main results

*Proof of Corollary 3.*

F , . -  
, $_1 \neq$ . I $_1 >$ ,  
$_H(_1) > _H()$, $_L(_1) > _L()$, $v_H(_1) < v_H()$, $v_L(_1) < v_L()$. H  
$)_1$ )

CR

. F $1 - \quad =$

$$\check{R} = N \left[ \begin{array}{l} \frac{\delta_H l_H}{k\, \delta l} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\, \delta l} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - (1 - \quad )) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - (1 - \quad )) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kL} y_{kL} \end{array} \right]$$

$$= N \left[ \begin{array}{l} \frac{\delta_H l_H}{k\, \delta l} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Hm}) y_{Hm} \\ + \frac{\delta_L l_L}{k\, \delta l} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(m)} (1 - \quad_{Lm}) y_{Lm} \\ + \frac{v_H q_H}{m\, vq} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kH} y_{kH} \\ + \frac{v_L q_L}{m\, vq} (1 - \quad ) E_{(k)} E_{(m)} y_{km} - E_{(k)} \quad_{kL} y_{kL} \end{array} \right]$$

$$\check{R} = N \quad 1 - ( \quad + \quad )$$

; - , . I ( L ). □

*Proof of Lemma 7.*

I , $k$

$$U_k = \; - c_w ( \;_k ) + \;_k M( \; )(1 - \;^w_k) w_k$$

$$= \; - c_w \; \frac{z^w_k}{M( \; ) w_k} \; + (1 - \;^w_k) z^w_k$$

$w_k$

$$\frac{@U_k}{@w_k} = -c'_w \frac{1}{M( \; ) w_k} \frac{@z^w_k}{@w_k} - c'_w \frac{z^w_k}{M( \; )} \; - \frac{1}{w^2_k} \; + \frac{@z^w_k}{@w_k} (1 - \;^w_k) = c'_w \frac{z^w_k}{M( \; ) w_k} > 0;$$

E . □

*Proof of Proposition 8.*

I: $w_k = E_{(m)} w_{km} = E_{(m)} \; {}_{km} y_{km}$

$k = H; L$; $z^w_k = \; {}_k M( \; ) w_k$

$k = H; L$; ${}_m = E_{(k)} \; {}_{km} = E_{(k)} (1 - \; {}_{km}) y_{km}$ fi

e $m = H; L$; $z^\pi_m = v_m{}^M$

Z

$\pi$ $\pi$

k

l . F fi , l , l

l , 

$$\frac{dz_H^w}{z_H^w}\quad \frac{1}{''^{w}_{H}} + (1-\quad)\frac{{}_H l_H}{k\quad l} \quad = (1-\quad)\quad E_{(m)}\quad \frac{dz_m^\pi}{z_m^\pi} \quad - \frac{{}_L l_L}{k\quad l}\quad \frac{dz_L^w}{z_L^w} \quad - \frac{d\quad {}^w_H}{1-\quad {}^\pi} \qquad ($$

L

[illegible]

$$\frac{dz^\pi_H}{z^\pi_H}\frac{1}{\varepsilon^\pi_H} = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k} + \frac{d\tau^\pi_L}{-\tau^\pi_L}\ \frac{v_L q_L}{m\, vq}\varepsilon^\pi_L - \frac{d\tau^\pi_H}{-\tau^\pi_H}\Big(1 + \frac{v_L q_L}{m\, vq}\varepsilon^\pi_L}{\Delta_2} \tag{45}$$

$$\frac{dz^\pi_L}{z^\pi_L}\frac{1}{\varepsilon^\pi_L} = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k} - \frac{d\tau^\pi_L}{-\tau^\pi_L}\Big(1 + \frac{v_H q_H}{m\, vq}\varepsilon^\pi_H + \frac{d\tau^\pi_H}{-\tau^\pi_H}\ \frac{v_H q_H}{m\, vq}\varepsilon^\pi_H}{\Delta_2}; \tag{46}$$

[illegible] $\Delta_2 = 1 + E_{(m)}\varepsilon^\pi_m$. A [illegible] (45) [illegible] (46) [illegible]

$$E_{(m)}\ \frac{dz^\pi_m}{z^\pi_m} = \frac{E_{(k)}\Big(\frac{dz^w_k}{z^w_k}\ E_{(m)}\varepsilon^\pi_m - E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_2}; \tag{47}$$

[illegible] (43) [illegible] (44) [illegible]

$$E_{(k)}\ \frac{dz^w_k}{z^w_k} = \frac{(1-\ )E_{(m)}\Big(\frac{dz^\pi_m}{z^\pi_m}\ E_{(k)}\varepsilon^w_k - E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k}}{\Delta_1}: \tag{48}$$

F [illegible] (47) [illegible] (48) [illegible] $E_{(m)}\Big(\frac{dz^\pi_m}{z^\pi_m}$ [illegible] $E_{(k)}\Big(\frac{dz^w_k}{z^w_k}$ [illegible]

$$E_{(m)}\ \frac{dz^\pi_m}{z^\pi_m} = -\frac{(\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1 + \Delta_2 - 1} \tag{49}$$

$$E_{(k)}\ \frac{dz^w_k}{z^w_k} = -\frac{\Delta_2 E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} + (\Delta_1 - 1)E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1 + \Delta_2 - 1} \tag{50}$$

[illegible] (49) [illegible] (43) [illegible] (44), [illegible] (50) [illegible] (45) [illegible] (46) [illegible]

$$\frac{dz^w_H}{z^w_H}\frac{1}{\varepsilon^w_H} = -\frac{(1-\ )\ (\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1(\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1)\ \ \frac{d\tau^w_L}{-\tau^w_L}(1-\ )\frac{\delta_L l_L}{k\,\delta l}\varepsilon^w_L - \frac{d\tau^w_H}{-\tau^w_H}\Big(1 + (1-\ )\frac{\delta_L l_L}{k\,\delta l}\varepsilon^w_L}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{51}$$

$$\frac{dz^w_L}{z^w_L}\frac{1}{\varepsilon^w_L} = -\frac{(1-\ )\ (\Delta_2 - 1)E_{(k)}\Big(\varepsilon^w_k \frac{d\tau^w_k}{-\tau^w_k} + \Delta_1 E_{(m)}\Big(\varepsilon^\pi_m \frac{d\tau^\pi_m}{-\tau^\pi_m}}{\Delta_1(\Delta_1 + \Delta_2 - 1)} + \frac{(\Delta_1 + \Delta_2 - 1)\ \ -\frac{d\tau^w_L}{-\tau^w_L}\Big(1 + (1-\ )\frac{\delta_H l_H}{k\,\delta l}\varepsilon^w_H + \frac{d\tau^w_H}{-\tau^w_H}(1-\ )\frac{\delta_H l_H}{k\,\delta l}\varepsilon^w_H}{\Delta_1(\Delta_1 + \Delta_2 - 1)}; \tag{52}$$

$$\frac{dz_H^\pi}{z_H^\pi}\frac{1}{''^{\pi}_H} = -\frac{\Delta_2 E_{(k)}\Big(''^{w}_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^{\pi}_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad \frac{d\tau_L^\pi}{-\tau_L^\pi} \quad \frac{v_L q_L}{_k\, vq}''^{\pi}_L - \frac{d\tau_H^\pi}{-\tau_H^\pi}\Big(1+ \quad \frac{v_L q_L}{_k\, vq}''^{\pi}_L}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{53}$$

$$\frac{dz_L^\pi}{z_L^\pi}\frac{1}{''^{\pi}_L} = -\frac{\Delta_2 E_{(k)}\Big(''^{w}_k \frac{d\tau_k^w}{-\tau_k^w} \quad + (\Delta_1-1)E_{(m)}\Big(''^{\pi}_m \frac{d\tau_m^\pi}{-\tau_m^\pi}}{\Delta_2(\Delta_1+\Delta_2-1)} + \frac{(\Delta_1+\Delta_2-1) \quad -\frac{d\tau_L^\pi}{-\tau_L^\pi}\Big(1+ \quad \frac{v_H q_H}{_k\, vq}''^{\pi}_H \quad + \frac{d\tau_H^\pi}{-\tau_H^\pi} \quad \frac{v_H q_H}{_k\, vq}''^{\pi}_H}{\Delta_2(\Delta_1+\Delta_2-1)}; \tag{54}$$

F (51)-(54) fi /.

$$\begin{aligned}
\frac{dz_H^w}{d\ {}^{w}_{H}}\frac{1}{z_H^w} &= \frac{''^{w}_H}{1-\ {}^{w}_{H}} \quad \frac{(1-\quad)\frac{\delta_H l_H}{_k\,\delta l}''^{w}_H - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^{w}_{H}}\frac{1}{z_L^w} &= \frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^{w}_L(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^{w}_{H}}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^{\pi}_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^{w}_{H}}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_H^w}{-\tau_H^w}\frac{\delta_H l_H}{_k\,\delta l}''^{\pi}_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{55}$$

$$\begin{aligned}
\frac{dz_H^w}{d\ {}^{w}_{L}}\frac{1}{z_H^w} &= \frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^{w}_H(1-\quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}^{w}_{L}}\frac{1}{z_L^w} &= \frac{''^{w}_L}{1-\ {}^{w}_{L}} \quad \frac{(1-\quad)\frac{\delta_L l_L}{_k\,\delta l}''^{w}_L - (\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^\pi}{d\ {}^{w}_{L}}\frac{1}{z_H^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^{\pi}_H}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^\pi}{d\ {}^{w}_{L}}\frac{1}{z_L^\pi} &= -\frac{\frac{\varepsilon_L^w}{-\tau_L^w}\frac{\delta_L l_L}{_k\,\delta l}''^{\pi}_L}{\Delta_1+\Delta_2-1}
\end{aligned} \tag{56}$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_H^{\pi}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}\,{}''^{w}_{H}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_H^{\pi}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}\,{}''^{w}_{L}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^{\pi}}{d\ {}_H^{\pi}}\frac{1}{z_H^{\pi}} &= \frac{{}''^{\pi}_{H}}{1- \ {}^{\pi}_{H}}\quad\frac{{}''^{\pi}_{H}\frac{v_H q_H}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^{\pi}}{d\ {}_H^{\pi}}\frac{1}{z_L^{\pi}} &= \frac{\frac{\varepsilon_H^{\pi}}{-\tau_H^{\pi}}\frac{v_H q_H}{{}_m vq}\,{}''^{\pi}_{L}}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{57}
$$

$$
\begin{aligned}
\frac{dz_H^w}{d\ {}_L^{\pi}}\frac{1}{z_H^w} &= -\frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}\,{}''^{w}_{H}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^w}{d\ {}_L^{\pi}}\frac{1}{z_L^w} &= -\frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}\,{}''^{w}_{L}(1- \quad)}{\Delta_1+\Delta_2-1}\\
\frac{dz_H^{\pi}}{d\ {}_L^{\pi}}\frac{1}{z_H^{\pi}} &= \frac{\frac{\varepsilon_L^{\pi}}{-\tau_L^{\pi}}\frac{v_L q_L}{{}_m vq}\,{}''^{\pi}_{H}}{\Delta_1+\Delta_2-1}\\
\frac{dz_L^{\pi}}{d\ {}_L^{\pi}}\frac{1}{z_L^{\pi}} &= \frac{{}''^{\pi}_{L}}{1- \ {}^{\pi}_{L}}\quad\frac{{}''^{\pi}_{L}\frac{v_L q_L}{{}_m vq}-(\Delta_1+\Delta_2-1)}{\Delta_1+\Delta_2-1}
\end{aligned}
\tag{58}
$$

[illegible]

I[illegible] L [illegible]

$$
\begin{aligned}
\max_{\tau_k^w,\tau_m^\pi} W = & \sum_k l_k \left(-c_w\left(\frac{z_k^w}{M(\ )\,w_k}\right)\right) + \sum_m q_m\left(-c_\pi\left(\frac{z_m^\pi}{\frac{M\,\theta}{\theta}\ {}_m}\right)\right)\\
& + {}_H l_H c_w'\left(\frac{z_H^w}{M(\ )\,w_H}\right) + {}_L l_L c_w'\left(\frac{z_L^w}{M(\ )\,w_L}\right) + v_H q_H c_\pi'\left(\frac{z_H^\pi}{\frac{M\,\theta}{\theta}\ {}_H}\right) + v_L q_L c_\pi'\left(\frac{z_L^\pi}{\frac{M\,\theta}{\theta}\ {}_L}\right) + R\\
& + (\ {}_k\ l)M(\ )\left(\frac{{}_H l_H}{{}_k\ l}\ {}^w_H w_H + \frac{{}_L l_L}{{}_k\ l}\ {}^w_L w_L + \frac{v_H q_H}{{}_m vq}\ {}^\pi_H\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}^\pi_L\ {}_L\right);
\end{aligned}
$$

[illegible] D[illegible]

$$
a = \frac{{}_H l_H}{{}_k\ l}\ {}^w_H w_H + \frac{{}_L l_L}{{}_k\ l}\ {}^w_L w_L \quad [illegible] \quad b = \frac{v_H q_H}{{}_m vq}\ {}^\pi_H\ {}_H + \frac{v_L q_L}{{}_m vq}\ {}^\pi_L\ {}_L:
$$

$$\frac{\partial L}{\partial \;{}_H^w} =$$

$$= \quad {}_k\, l_k \left( -\frac{c_w'}{M(\;)\,w_k}\frac{dz_k^w}{d\;{}_H^w} \right) + {}_m\, q_m \left( -\frac{c_\pi'}{\frac{M\ \theta}{\theta}\;{}_m}\frac{dz_m^\pi}{d\;{}_H^w} \right)$$

$$+\frac{dz_H^w}{d\;{}_H^w}\frac{1}{M(\;)\,w_H} l_H c_w' \quad \frac{z_H^w}{M(\;)\,w_H} \quad + \quad {}_H l_H c_w'' \quad \frac{z_H^w}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H}\frac{dz_H^w}{d\;{}_H^w}$$

$$+\frac{dz_L^w}{d\;{}_H^w}\frac{1}{M(\;)\,w_L} l_L c_w' \quad \frac{z_L^w}{M(\;)\,w_L} \quad + \quad {}_L l_L c_w'' \quad \frac{z_L^w}{M(\;)\,w_L} \quad \frac{1}{M(\;)\,w_L}\frac{dz_L^w}{d\;{}_H^w}$$

$$+\frac{dz_H^\pi}{d\;{}_H^\pi}\frac{1}{\frac{M\ \theta}{\theta}\;{}_H} q_H c_\pi' \quad \frac{z_H^\pi}{\frac{M\ \theta}{\theta}\;{}_H} \quad + v_H q_H c_\pi'' \quad \frac{z_H^\pi}{\frac{M\ \theta}{\theta}\;{}_H} \quad \frac{1}{\frac{M\ \theta}{\theta}\;{}_H}\frac{dz_H^\pi}{d\;{}_H^\pi}$$

$$+\frac{dz_L^\pi}{d\;{}_L^\pi}\frac{1}{\frac{M\ \theta}{\theta}\;{}_L} q_L c_\pi' \quad \frac{z_L^\pi}{\frac{M\ \theta}{\theta}\;{}_L} \quad + v_L q_L c_\pi'' \quad \frac{z_L^\pi}{\frac{M\ \theta}{\theta}\;{}_L} \quad \frac{1}{\frac{M\ \theta}{\theta}\;{}_L}\frac{dz_L^\pi}{d\;{}_L^\pi}$$

$$+\left[ {}_k \frac{l_k}{M(\;)\,w_k}\frac{dz_k^w}{d\;{}_H^w} \quad M(\;) + (\;{}_k\, l)M(\;) \left( \frac{{}_m \frac{q_m}{\frac{M(\theta)}{\theta}\pi_m}\frac{dz_m^\pi}{d\tau_H^w}}{{}_k\, l} - \frac{(\;{}_m\, vq)\;{}_k\left(\frac{l_k}{M\ \theta\ \ w_k}\frac{dz_k^w}{d\tau_H^w}\right)}{(\;{}_k\, l)^2} \right) \right] (a+b)$$

$$+\;(\;{}_k\, l)M(\;)$$

fi

$$ {}_{H} l_H \, c''_w \; \frac{z^w_H}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H}\frac{dz^w_H}{d\;{}^w_H} + \; {}_{L} l_L \, c''_w \; \frac{z^w_L}{M(\;)\,w} $$ [redacted]

$$ +v_H q_H \, c''_\pi \; \frac{z^\pi_H}{\frac{M\;\theta}{\theta}\;\;{}_H} \quad \frac{1}{\frac{M\;\theta}{\theta}\;\;{}_H}\frac{dz^\pi_H}{d\;{}^w_H} + v_L q_L \, c''_\pi \; \frac{z^\pi_L}{\frac{M\;\theta}{\theta}\;\;{}_L} $$ [redacted]

$$ + \;(\;\;{}_k\;\; l)M(\;)\left[\; \frac{{}_k\left(\frac{l_k}{M\;\theta\;\;w_k}\frac{dz^w_k}{d\tau^w_H}\right.}{{}_k\;\;l} + \frac{M'(\;)}{M(\;)}\;\;\frac{m}{} \right. $$ [redacted] $$ \left. \frac{\frac{l_k}{M\;\theta\;\;w_k}\frac{dz^w_k}{d\tau^w_H}}{{}_k\;\;l} \right] \; (a+b) = $$

$$ = \;\; {}_{H} l_H \, c''_w \; \frac{z^w_H}{M(\;)\,w_H} \quad \frac{1}{M(\;)\,w_H}\frac{dz^w_H}{d\;{}^w_H}\frac{z^w_H}{z^w_H}\frac{M(\;)\,(}{c'_w(z^w_H=} $$ [redacted]

$$ + \;\; {}_{L} l_L \, c''_w \; \frac{z^w_L}{M(\;)\,w_L} \quad \frac{1}{M(\;)\,w_L}\frac{dz^w_L}{d\;{}^w_H}\frac{z^w_L}{z^w_L}\frac{M(\;)\,(1-\;\;{}^w_L)w_L}{c'_w(z^w_L=M(\;)\,w_L)} $$

$$ +v_H q_H \, c''_\pi \; \frac{z^\pi_H}{\frac{M\;\theta}{\theta}\;\;{}_H} \quad \frac{1}{\frac{M\;\theta}{\theta}\;\;{}_H}\frac{dz^\pi_H}{d\;{}^w_H}\frac{z^\pi_H}{z^\pi_H}\frac{M(\;)}{}\frac{(1-\;\;{}^\pi_H)\;\;{}_H}{c'_\pi(z^\pi_H=\;\;{}_H)} $$

$$ +v_L q_L \, c''_\pi \; \frac{z^\pi_L}{\frac{M\;\theta}{\theta}\;\;{}_L} \quad \frac{1}{\frac{M\;\theta}{\theta}\;\;{}_L}\frac{dz^\pi_L}{d\;{}^w_H}\frac{z^\pi_L}{z^\pi_L}\frac{M(\;)}{}\frac{(1-\;\;{}^\pi_L)\;\;{}_L}{c'_\pi\left(z^\pi_L=\frac{M\;\theta}{\theta}\;\;{}_L\right.} $$

$$ + \;(\;\;{}_k\;\; l)M(\;)\left[\;\; {}_k\left(\frac{l_k}{M\;\theta\;\;u\text{wH}}\right.\right. $$

$$= \ {}_{H} l_{H} \frac{1}{''^{w}_{H}} \frac{dz^{w}_{H}}{d \ {}^{w}_{H}} \frac{1}{z^{w}_{H}}$$

$_{H}^{\pi}$, $_{L}^{\pi}$

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad)^{1 - \quad {}_{L}^{w}}$$

F

$$= \quad 1 - \frac{\left[ \begin{array}{l} \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_L q_L}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_H q_H}{{}_m vq}\right) w_{HH} + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_H q_H}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_L q_L}{{}_m vq}\right) w_{HL} \quad {}^{w}_{H} \\ + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_H q_H}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_L q_L}{{}_m vq}\right) w_{LH} + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_L q_L}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_H q_H}{{}_m vq}\right) w_{LL} \quad {}^{w}_{L} \\ + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_L q_L}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_H q_H}{{}_m vq}\right) {}_{HH} + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_H q_H}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_L q_L}{{}_m vq}\right) {}_{HL} \quad {}^{\pi}_{H} \\ + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_H q_H}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_L q_L}{{}_m vq}\right) {}_{LH} + \left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_L q_L}{{}_m vq} + \frac{\delta_L l_L}{{}_k \delta l} \frac{v_H q_H}{{}_m vq}\right) {}_{LL} \quad {}^{\pi}_{L} \end{array} \right]}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_L} \quad ;$$

$$= \left( 1 - \frac{E_{(s)} w_{Hs} \, {}^{w}_{H} + E_{(s)} w_{Ls} \, {}^{w}_{L} + E_{(s)} \, {}_{sH} \, {}^{\pi}_{H} + E_{(s)} \, {}_{sL} \, {}^{\pi}_{L}}{\frac{-\tau^w_H}{\varepsilon^w_H} w_H + \frac{-\tau^w_L}{\varepsilon^w_L} w_L + \frac{-\tau^\pi_H}{\varepsilon^\pi_H} {}_H + \frac{-\tau^\pi_L}{\varepsilon^\pi_L} {}_L} \right) ; \qquad (64)$$

$s$ $\left(\frac{\delta_H l_H}{{}_k \delta l} \frac{v_L q_L}{{}_m vq} + \ldots\right.$

$c'$

$$'' = \frac{1}{\;} = \frac{c''}{c'} = \frac{1}{\; - 1}.$$

, $A$, :

.

**2** $c = A( \; ^{\gamma} + \; ^{\beta})$ $\geq 3$, $<$ , $c' = A( \; ^{\gamma-} + \; ^{\beta-})$,

$$c'' = A( \; ( \; - 1) \; ^{\gamma-2} + \; ( \; - 1) \; ^{\beta-2}) > 0;$$

$$'' = \frac{\; ^{\gamma-2} + \; ^{\beta-2}}{( \; - 1) \; ^{\gamma-2} + ( \; - 1) \; ^{\beta-2}}.$$

$A$ , , .

$$\frac{@''}{@} = - \; ^{\beta} \; ^{\gamma-} ( \; - \; )^2 < 0.$$

N $\geq 3$ $= \; - 1$. . F , $< 1$ $= 3$ $= 2$, $> 1$ $= 2$ $= 1$.

$\frac{-\tau_H^w}{\varepsilon_H^w} w_H < \frac{-\tau_L^w}{\varepsilon_L^w} w_L$, (66) -. H $(1 - \; _H^w) w_H < (1 - \; _L^w) w_L$ $\; _H^w > \; _L^w$ (66) $\; _H^w w_H > \; _L^w w_L$. N $(E_{(m)} \; _{Hm} \; _m^{\pi} - E_{(m)} \; _{Lm} \; _m^{\pi})$. - $\; _{HH} > \; _{LH}$ $\; _{HL} > \; _{LL}$ $(E_{(m)} \; _{Hm} \; _m^{\pi} - E_{(m)} \; _{Lm} \; _m^{\pi} > 0)$. H (66) . $(1 - \; _H^w) w_H > (1 - \; _L^w) w_L$, $\; _H > \; _L$, $''^w_H \leq ''^w_L$. □

*Proof of Proposition 11.*

fi (65). fi . , , (60)-(63) . fi , . D

(60) (61)

$$(\Delta_1 + \Delta_2 - 1) \quad (1 - \quad) \frac{1 - \quad ^{w}}{\quad ''_{w}} w + \quad (w \quad ^{w} + \quad ^{\pi} - (1 - \quad) \bar{R}) \quad =$$

$$= (1 - \quad) \left[ (1 - \quad ^{w}) w + \quad (w \quad ^{w\,''w} + \quad ^{\pi\,''w} - (1 - \quad) \bar{R}^{\,''w}) \right]$$

$$-$$

(68), (69), P 11.

P 12

(1 − ), (1 − )= ↑,

$^{\pi}$= $^{w}$ ↓.□