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Estimating Discount Factors within a Random Utility Theoretic Framework

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Estimating Discount Factors within a Random Utility Theoretic Framework*

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Abstract

Choices involving tradeo¤s of bene…ts and costs over time are pervasive in our everyday lives. The observation of declining discount rates in experimental settings has led many to promote hyperbolic discounting over standard exponential discounting as the preferred descriptive model of intertemporal choice. In this paper, I develop a new framework that directly models the intertemporal utility function associated with an intertemporal outcome. This random utility model produces explicit maximum likelihood estimates of the discounting parameters. The main bene…t of this approach is that I am able to perform formal statistical tests of quasi-hyperbolic and hyper-

1 Introduction

Every day we make decisions involving tradeoffs of benefits and costs over time. Would I rather spread my workload evenly over the next few days and distribute the pain or procrastinate and have an extremely painful task several days from now? Should I exercise regularly while I'm younger so that I can enjoy the health benefits when I'm older? Will I invest time and money in my education today so that I can have a better lifestyle later? Am I willing to give up some consumption today so that I and others can enjoy a better environment in the future? These intertemporal choices penetrate nearly every aspect of our behavior. Such decisions the model on three data sets. Two of the data sources are original; one comes from a stated-preference survey on river basin improvements and one comes from choices simplicity and elegance. Interestingly, Samuelson did not endorse the DU model as a normative model of intertemporal choice or as a valid descriptive model. The DU model was never empirically veri…ed but still became the standard model for intertemporal utility. [24]

2.2 Departures from the Discounted Utility Model

In the past several decades, research has uncovered many situations in which the DU model does not …t behavior.[2] One major departure from the DU model is that inferred discount rates often decline over time in experimental settings. This phenomenon is commonly termed hyperbolic discounting. This discounting gets its name because a hyperbolic functional form …ts the data better than the traditional exponential functional form. Several functional forms have been suggested for hyperbolic discounting. The most popular of these takes the form of

$$_t = (1 + \quad t)^{\quad =} \text{; where } \quad ; \quad > 0 \text{ [19]:} \qquad (2)$$

As

In recent years, an alternative model of discounting that has received much attention is the quasi-hyperbolic (;) discounting model. This model, developed by David Laibson, is also motivated by the observation of declining discount rates [18]. The functional form was first introduced by Phelps and Pollak in the context of intergenerational altruism [22]. The form of the quasi-hyperbolic discounting function is very simple and its contrast with the standard exponential discounting model is readily apparent. The functional form is given by

$$ {}_t = \begin{cases} 1 & \text{if } t = 0 \text{ and} \\ {}^t & \text{if } t > 0 \end{cases} ; \text{ where } 0 < \quad < 1; \text{ and } \quad < 1: \qquad (5) $$

Thus, the only difference between discount factors in the quasi-hyperbolic formulation and the exponential formulation is that all future time periods are discounted by the additional factor in the quasi-hyperbolic model. Especially large importance is placed on immediate utility as compared to deferred utility. The (;) discounting model is much easier to analyze than the true hyperbolic model, yet it retains many of the qualitative aspects of the more complicated model.

As shown in Figure 1 [3], both hyperbolic and the quasi-hyperbolic discounting functions weight the near future less heavily than exponential discounting. However, for time periods far in the future, exponential discounters place less weight on the deferred utility than hyperbolic or quasi-hyperbolic discounters. Figure 2 shows the corresponding marginal discount rates for all four discounting functions. The point plotted for time period t is the marginal discount rate between time period $t - 1$ and time period t:

[3] The parameter values used for the exponential, Harvey hyperbolic, and HM hyperbolic models in these figures are consistent with those that I find from the data sets employed in this paper. The chosen for the quasi-hyperbolic model is in the range of values discussed in the literature.

Figure 2: Comparison of Marginal Discount Rates: Exponential (^{t}) with = :9, Harvey Hyperbolic ((1 + t)) with = :4, Quasi-hyperbolic (1; ^{t}) with = :75, = :92, and HM Hyperbolic ((1 + ! t) ^{1}) with ! = :15.

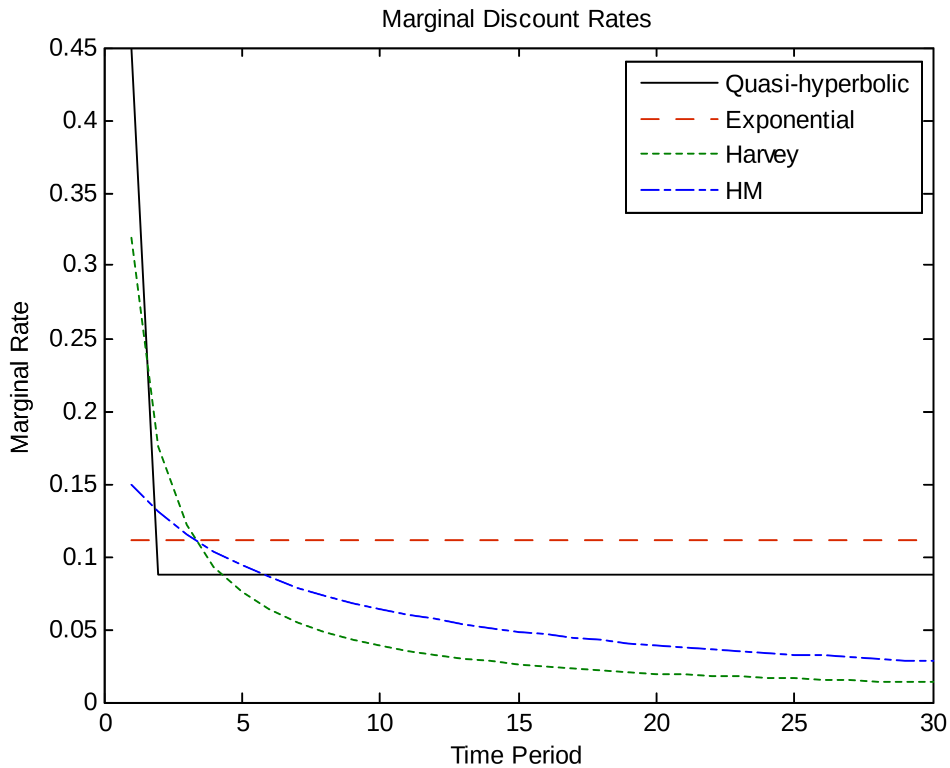


Table 1: Present Discounted Values of 100 Dollars Now vs. 120 Dollars 1 Year from Now for Exponential with = :9, Harvey Hyperbolic with = :4, HM Hyperbolic with ! = :15, and Quasi-hyperbolic with [illegible]

2.3 Discounting Studies

I concentrate on several of the more recent contributions and note that a more extensive literature review on discounting is provided by Frederick, Loewenstein, and O'Donoghue [8]. Table 3 summarizes several of the discounting studies related to public goods. Table 4 provides examples of the more common money discounting studies. While three recent working papers use utility-theoretic models incorporating goods other than money, the majority of previous studies examine monetary trade-offs over time. Table 5 summarizes some of the indirect tests on various discounting models. I point out that much of the evidence supporting hyperbolic discounting can be recast in terms of confounding factors. I am not aware of any previous research that has performed direct nested testing like I propose in this research.

Table 3: Empirical Discounting Studies (Health and Public Goods)

Author	Type	Discounting	Methodology	Good	Time Frame
Cameron, Gerdes[6]	experimental	exponential, hyperbolic	RUM, money lottery combined with conjoint		

Table 4: Empirical Discounting Studies (Money)

Author	Type	Discounting	Methodology	Good	Time Frame
Warner, Pleeter[30]	revealed	exponential	Lump-sum or annuity choice during downsizing, reduced form probit	money	0 to 2 times years of service
Coller, Williams[7]	experimental	exponential	choice between payo¤ now and later, censored data, maximum likelihood	money	0 to 3 months
Harrison, Lau, Williams[9]	experimental	exponential	choice between payo¤ now and later, individual explanatory variables	money	0 to 36 months
Alberini,					

Table 5: Indirect Tests of Discounting Models

Author	Type	Discounting	Methodology	Good	Time Frame
Kirby and Marakovic[17]	experimental	exponential and hyperbolic	matching task, …t exponential and hyperbolic parameters for each subject	money	3 to 29 days
Slonim et al.[25]	experimental	Informally tests whether discount rates are di¤erent for longer front-end delays. Compares patterns to exponential, hyperbolic, and quasi-hyperbolic models.	Discrete choices between earlier and later payo¤. Reduced form regression on decision to wait.	money	0 to 6 months
Cairns and van der Pol[4]	experimental	Compare 3 hyperbolic models with exponential	Choice between bene…t 1 year from waitJET100190682.178cmq8x-py		

2.3.1 Estimation Methods

The most common method for gathering data on discounting is to elicit experimental responses to hypothetical or real monetary rewards. Two approaches are most widely reduce the number of illnesses and deaths in their community. At the same time, they have individuals choose between a hypothetical lottery that provides a series of payments over several years and a lottery that provides a lump sum payment. This method is based on the identification strategy developed by Cameron and Gerdes [6]. The authors of both papers argue that the two distinct data sources allow improved joint estimation of the utility parameters and discount rates and that it is often not possible to identify discounting parameters out of a public goods choice.

I show that discounting parameters for public goods are identified in a stated-

2.3.2 Confounding Factors in Discounting Studies

Although evidence in the literature suggests that individuals have hyperbolic discounting preferences, I propose that much of this evidence can be explained by confounding factors. As emphasized in the review article by Frederick, Loewenstein, and O'Donoghue [8], it is important to di¤erentiate between pure rates of time preference and other reasons that cause individuals to care less about future outcomes. Pure time preference refers to "the preference for immediate utility over delayed utility" [8]. Confounding factors that cause individuals to care less about the future but should be considered separately from pure time preference include uncertainty about a future outcome, perceived future transaction costs, and the phenomenon of subadditive discounting. In this section, I show how experimental designs that do not address these three confounding factors could make an exponential discounter appear as though they are a hyperbolic discounter.

Imagine an experimental setting in which an individual is choosing between a smaller immediate reward and a larger delayed reward. Uncertainty in the receipt of the future reward can be problematic for estimating discount rates in this scenario. Suppose that this individual is truly an exponential discounter but perceives only a 70 percent chance that the researcher will actually deliver the delayed reward at any time in the future and a 100 percent chance that the immediate reward will be delivered. Then, the results from the experiment would look exactly like the individual is a quasi-hyperbolic discounter with a value of 0.7. Or, suppose that this individual is truly an exponential discounter with a constant discount factor of < 1 but believes with probability $p_0 = 1$ that they will receive an immediate reward, with probability $p_1 < 1$ that they will receive a delayed reward at $t = 1$, and with probability p_t, such that $p_{t+1} < p_t$ and $p_{t+1} - p_t > p_{t+2} - p_{t+1}$, that they will receive a delayed reward at time t. That is, the perceived probability of receiving a future reward declines at a decreasing rate. Then, observed discount factors including the confounding e¤ect of uncertainty are given by $\{1; p_1 \ ; p_2 \ ^2; p_3 \ ^3; p_4 \ ^4; :::\}$: Marginal observed discount rates are given by $\{1=p_1 \ - 1; p_1= \ p_2 - 1; p_2= \ p_3 - 1; p_3= \ p_4 - 1; :::\}$: These resulting observed discount rates are consistent with a hyperbolic functional form. To further illustrate with a numerical example, assume $\ = :9; p_1 = :8; p_2 = :7; p_3 = :65; p_4 = :61$: This gives marginal discount rates of $\{38{:}9\%; 26{:}9\%; 19{:}7\%; 18{:}4\%\}$: However, when abstracting from the e¤ects of uncertainty, true marginal rates of time preference are given by $\{1= \ - 1; 1= \ - 1; 1= \ - 1; :::\}$. Thus, it is important to minimize the e¤ects of future uncertainty in a discounting study.

Next, suppose that within an experimental setting an individual perceives a transaction cost of c_t in order to collect a payment at time t in the future. Also suppose that this individual is an exponential discounter with a discount factor of $\ ^t$. Then, in order to be indi¤erent between an immediate payment of $\$x_0$ and a delayed payment of $\$x_t$; it must be that $x_0 = \ ^t(x_t + c_t)$: If $c_{t+1} = c_t$ for all $t > 0$, observed marginal discount rates look like quasi-hyperbolic discount rat.748-2d[(211(e)9(.)-786(A)16(I)97-1.55Tf11.864

the e¤ect of the transaction cost, 100 = ${}_{2}^{obs^2}$(123:46 + 20): Solving, ${}_{2}^{obs^2}$ = :6971. This implies $r_{1;2}^{obs}$ = ${}_{2}^{obs^2}$= ${}_{1}^{obs}$ $-$ 1 = 18:45%: Continuing with this pattern, I …nd $r_{2;3}^{obs}$ = 16:53% and $r_{3;4}^{obs}$ = 15:10%. I observe declining marginal discount rates even though the true marginal discount rates are constant. The larger the transaction cost relative to the size of the reward, the more pronounced this e¤ect will be.

One other explanation for the observation of declining discount rates is the idea of subadditive discounting. That is, "discounting over a delay is greater when the delay is divided into subintervals than when it is left undivided" [23]. Most laboratory experiments look over days or months and confound the length of the delay with the each subject. They utilize nonlinear regression techniques on the continuous time equations for exponential and hyperbolic discounting. They find that, while both do a good job explaining subjects' responses, the hyperbolic model fits better in terms of R^2 for almost all of the subjects. Uncertainty in the payment of the delayed reward is present since delayed rewards were not to be delivered until the evening on the day that it came due. Transaction costs are especially relevant because the rewards are small ($14.75-$28.50 for delayed rewards). This study confounds length of delay until the delayed reward is received with the length of the interval between options since all choices are anchored to the present.

Slonim et al. [25] conduct an experimental study in which they examine whether or not possession of the delayed reward affects subjects' discounting patterns. They find that discount rates decline over time in all cases. Possession of rewards supports quasi-hyperbolic discounting and no possession supports hyperbolic discounting. They do not find any evidence of exponential discounting. This study attempts to control for transaction costs in the best way possible by using possession of the reward as a control variable. Also, this study uses a common interval length of two months for all choices so interval length is not confounded with the length of delay until the receipt of the future reward. Uncertainty in future rewards is nullified in the cases where individuals choose between two future rewards if the perceived probability of receipt of the reward is constant over time. However, uncertainty in future rewards is still an issue if the probability of receipt of the reward declines with longer time delays. Also, for the choices anchored to the present, uncertainty in future rewards remains a confounding factor.

Cairns and van der Pol [4] compare three hyperbolic models with the exponential model. For each individual and discounting model, they first estimate optimal parameter values using non-linear least squares. Second, they regress these parameter values on the period in years for which the benefit is delayed, claiming that delay should be insignificant for a correctly specified discounting model. Delay is insignificant only in the Loewenstein and Prelec model (2 parameter hyperbolic). They also note that the first stage regressions have the highest R^2 for the hyperbolic models. Since all choices are anchored to one year in the future, uncertainty in rewards is controlled for if the perceived probability of receiving the reward is constant over all time periods but not if the perceived probability of receipt declines with time. Transaction costs are minimized in the case of social financial benefits since the receipt of the reward does not require any work on part of the survey respondent. For private financial benefits, transaction costs likely get larger as the delayed reward moves farther into the future. If transaction costs are constant over all future time periods, they will have no influence in this study since all choices are anchored to one year from the present. However, because of this common anchor, the length of delay and length of interval are confounded. Subadditive discounting may explain any evidence for hyperbolic discounting.

Keller and Strazzera [16] examine the predictive accuracy of the exponential and hyperbolic models in a simulated data set. Using Thaler's [26] 1981 experimental data to calculate implicit monthly discount rates, the authors generate a simulated jointly addressing these experimental concerns and developing a new empirical model that directly estimates the discounting parameters, I am able to isolate pure rates of time preference for various models and test to find the statistically preferred model.

3 Empirical Strategy

3.1 Derivation of the General Model

Here I present the random utility model to analyze discrete choice data. This model analyzes choices over goods that are intertemporal in nature. In general, let the instantaneous utility for an individual i for choice j in year t be given by

$$u_{ijt} = v_{ijt} + \epsilon_{ijt}. \tag{6}$$

Here, v_{ijt} is the deterministic portion of utility and ϵ_{ijt} is the instantaneous error draw. It is important to note at this point that instantaneous utility is not at all observable. That is, the researcher only observes behavior at the choice level.

I make the usual assumption that intertemporal utility is additively separable over time periods. Then the utility for individual i that is associated with choice j defined through time period T_j is given by

$$U_{ij}(u_{ijt}; \delta_t) = \delta_0 u_{ij0} + \delta_1 u_{ij1} + \cdots + \delta_{T_j} u_{ijT_j}; \tag{7}$$

where δ_t is the discount factor for year t. Substituting equation 6 into equation 7 and rewriting in summation notation produces

$$U_{ij} = \sum_{t=0}^{T_j} \delta_t v_{ijt} + \epsilon_{ij}; \tag{8}$$

where $\epsilon_{ij} = \sum_{t=0}^{T_j} \delta_t \epsilon_{ijt}$ is the error for individual i associated with choice j. Thus, the intertemporal utility from a choice is essentially the weighted sum of all instantaneous utilities. Discount factors determine the weight placed on each time period. The speci…cation of v_{ijt} will depend on the type of intertemporal choice that is being analyzed.

3.2 Structure of the Error Terms

Since a rational individual makes utility evaluations at the instantaneous level and discounts them back to the present, it is appropriate to assume the distribution of the instantaneous errors ($_{ijt}$). However, the researcher observes choices at the alternative level so it is necessary to use the model structure to determine the alternative level error structure. This approach contrasts the Bosworth et al. assumption that alternative errors are i.i.d. extreme value. I show in this section that even i.i.d. error assumptions at the instantaneous level imply heteroskedastic errors at the alternative level.

I …rst examine the expectation of the alternative error terms and then explore the alternative error variance structure.

Proposition 1 The alternative roa8tro sre(t)8r)12i)6b)11(u)11(1411.5Tf0 Td[(P)1.42Td17)8e)8r)12v)8e)1

Proof. $V(\ _{ij}) = V(\sum_{t=0}^{T_j}\ _t\ _{ijt})$

$= V(\ _0\ _{ij0} + \cdots + \ _{ijT_j})$

$= \ ^2_0 V($

The task is to determine the form of P_{ij}. Begin by substituting equation 8 into equation 9 to get

$$P_{ij} = \Pr\left(\sum_{t=0}^{T_j} \beta_t v_{ijt} + \varepsilon_{ij} > \sum_{t=0}^{T_k} \beta_t v_{ikt} + \varepsilon_{ik}\right) \tag{10}$$

$$= \Pr\left(\varepsilon_{ik} - \varepsilon_{ij} < \sum_{t=0}^{T_j} \beta_t v_{ijt} - \sum_{t=0}^{T_k} \beta_t v_{ikt}\right) \tag{11}$$

Next, denote the alternative error-difference term as $\tilde{\varepsilon}_{ikj} = \varepsilon_{ik} - \varepsilon_{ij}$. Recalling that $\varepsilon_{ij} = \sum_{t=0}^{T_j} \beta_t \eta_{ijt}$, I have

$$\tilde{\varepsilon}_{ikj} = \sum_{t=0}^{T_k} \beta_t \eta_{ikt} - \sum_{t=0}^{T_j} \beta_t \eta_{ijt}. \tag{12}$$

For any decision maker, i, and time period, t, assume that if $v_{ijt} = v_{ikt}$, then $\eta_{ijt} = \eta_{ikt}$. That is, within a time period, if the observable components of utility associated with two choices for a given decision maker are equal, then the instantaneous error draws are equal also. For this analysis, assume that there are no time periods for which the observable components of utility are exactly the same. Then, note that $\tilde{\varepsilon}_{ikj}$ is heteroskedastic because the number of terms in the summations is determined by the length of the intertemporal alternative. $\tilde{\varepsilon}_{ikj}$ is a normal error term with mean zero and variance given by

$$V(\tilde{\varepsilon}_{ikj}) = V\left(\sum_{t=0}^{T_k} \beta_t \eta_{ikt} - \sum_{t=0}^{T_j} \beta_t \eta_{ijt}\right). \tag{13}$$

$$= \beta_0^2 V(\eta_{ik0}) + \beta_1^2 V(\eta_{ik1}) + \ldots + \beta_{T_k}^2 V(\eta_{ikT_k}) + \beta_0^2 V(\eta_{ij0}) + \beta_1^2 V(\eta_{ij1}) + \ldots + \beta_{T_j}^2 V(\eta_{ijT_j}) \tag{14}$$

since the instantaneous errors are independent. With the assumption that ϵ_{ijt} i.i.d $N(0;\sigma)$. This leads to

$$V(\tilde{\epsilon}_{ikj}) = \sum_{t=0}^{T_k} \sigma_t^2 + \sum_{t=0}^{T_j} \sigma_t^2 . \tag{15}$$

It is well known that a probit model needs to be normalized for scale so set $\sigma = 1$ and I have

$$V(\tilde{\epsilon}_{ikj}) = \sum_{t=0}^{T_k} \sigma_t^2 + \sum_{t=0}^{T_j} \sigma_t^2 = V(\epsilon_{ik}) + V(\epsilon_{ij}). \tag{16}$$

Therefore, for any choice set, the variance of the alternative error-difference term will be larger when both policies have longer durations. Ignoring this in the likelihood function will lead to inconsistent parameter estimates and biased standard error estimates. Returning to equation 10 and using the definition of the c.d.f. (F) of a normal random variable, I have

$$P_{ij} = F\left(\frac{\sum_{t=0}^{T_j} \beta^t v_{ijt} - \sum_{t=0}^{T_k} \beta^t v_{ikt}}{\sqrt{\sum_{t=0}^{T_k} \sigma_t^2 + \sum_{t=0}^{T_j} \sigma_t^2}} \right). \tag{17}$$

The log-likelihood equation is then

$$LL = \sum_i \sum_j y_{ij} \ln P_{ij}; \tag{18}$$

where $y_{ij} = 1$ if i chose alternative j and zero otherwise.

Note that observations from choice sets with alternatives having longer durations are weighted less heavily than observations from choice sets with alternatives having shorter durations. Again, this serves a control for potential decision-maker uncertainty. Observations associated with longer time dimensions likely have more confounding effects from uncertainty so they receive less weight in the likelihood function.

3.4 Application to a Public Good Choice

This model is particularly well suited to analyze attribute based stated-preference data. Attribute based (conjoint) survey designs allow the researcher to specify several attribute dimensions of the intertemporal choices. Thus, the researcher can specify when the benefits and costs of an intertemporal choice are to be realized so that it is possible to identify the discount factors from respondents' choices. Public goods policies are a good example of choices that receive benefits and costs at differing points in times. For example, it is common to pay taxes today for a public good that will deliver benefits years into the future. In this section I develop the model for conjoint data in the context of public goods choices.[6]

At any time the utility an individual receives from a simple public good policy depends on the level of benefit provided and the cost incurred. Specify the deterministic portion of instantaneous utility as

$$v_{ijt} = \quad q_{ijt} + \quad (Y_{it} - c_{ijt}); \tag{19}$$

where q_{ijt} is the level of benefits from the public good, Y_{it} is income, and c_{ijt} is the cost of the public good for individual i for policy j in year t. In this specification, is the marginal utility of the public good benefit and is the marginal utility of money. Let T_j denote the last year for which there are non-zero costs or benefits for policy j. Substituting equation 19 into equation 8 results in

$$U_{ij} = \sum_{t=0}^{T} \ _t[\ q_{ijt} + \ (Y_{it} - c_{ijt})] + \ _{ij}: \tag{20}$$

This equation is the foundation of my econometric model.

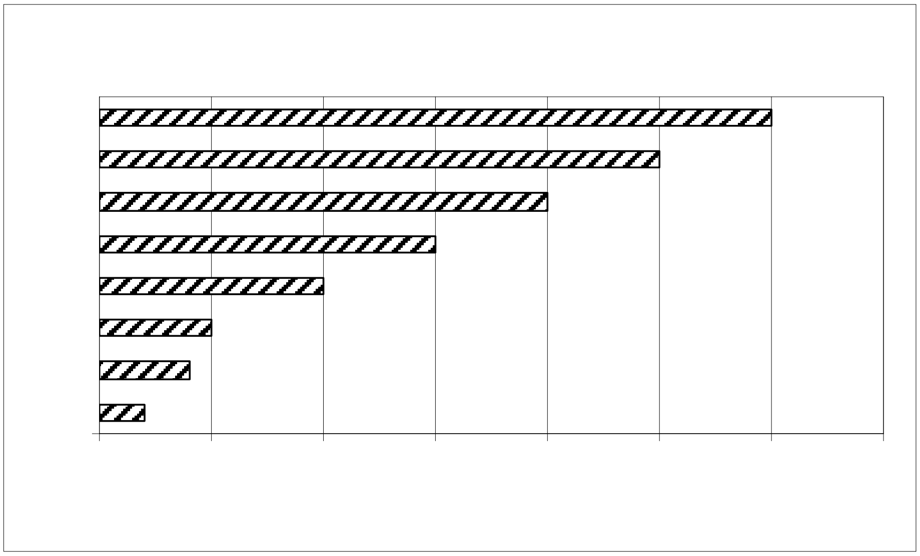
Because only differences in utility matter in the RUM, any personal characteristic

[6] Viscusi and Huber [29] provide the first example of a study designed to infer discount rates for public goods.

on its own such as Y_{it} drops out of the analysis. Personal characteristics can enter through interactions with policy characteristics. Since $_0$ = 1 by economic theory, there are T_j + 2 parameters to estimate in this model. The parameter is identi...ed deterministic portion of instantaneous utility as

$$v_{ijt} = \quad (Y_{it} + m_{ijt});$$

Figure 3: Survey Design 1



Huber design their survey such that costs uniformly begin immediately and run for five years [29]. Benefits (improvements to local water quality) begin with a delay of 0, 2, 4, or 6 years and run for five years. After five years, the water quality returns to the status quo at the beginning of the policy. In this design, per-year costs and benefits are also constant throughout the duration of the policy. (See Figures 4,5.) The following figures show eight different hypothetical policies for each survey design. The shaded boxes represent the duration of the various policies.

Figure 4: Survey Design 2 Bene...ts

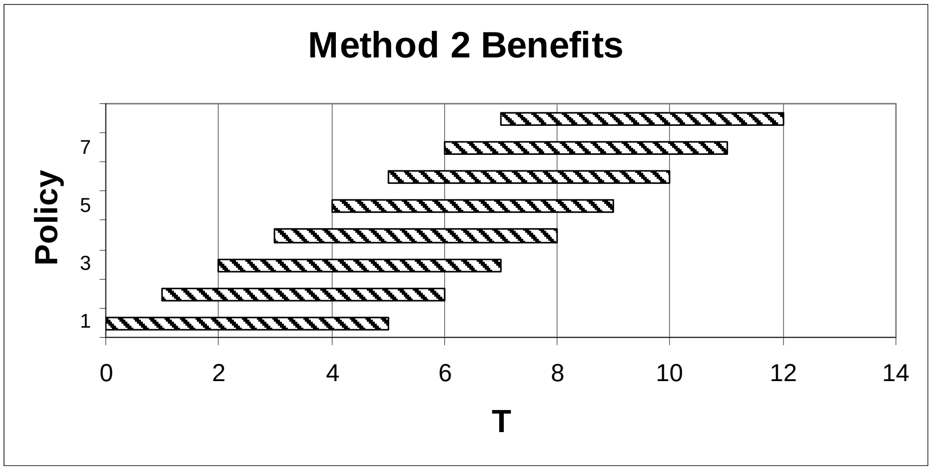
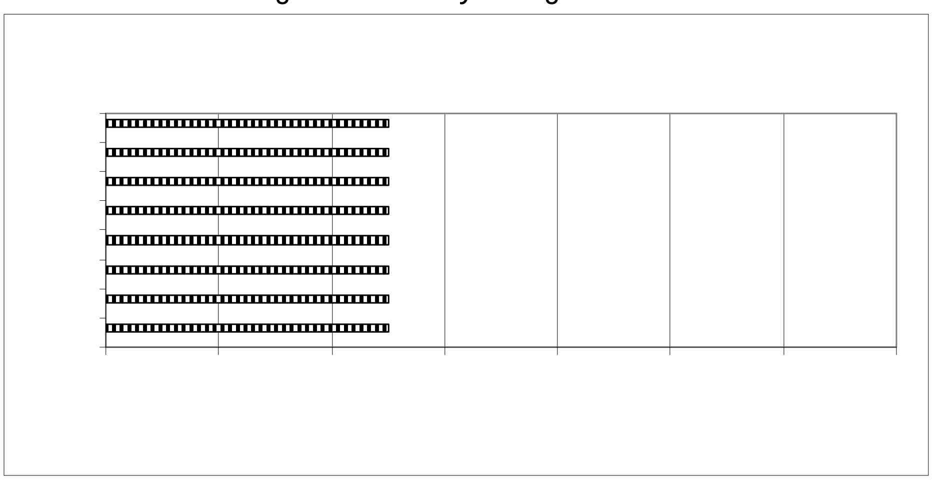


Figure 5: Survey Design 2 Costs



When considering the design of the survey it is important to make the policy choices as close to a real life situation as possible. In the case of public goods, I believe that it is most realistic to have costs uniformly start today and benefits start with a delay of zero to Y years, with Y selected such that respondents still believe that the policy will affect them. It is common for taxes to begin now and continue with a specific duration at the same cost per year and benefits to arrive at different times in the future at the same level of benefits per year. Therefore, I design my survey like "Method 2."

There are four principles identified as important in the literature when designing survey questions for choice experiments (conjoint questions). Level balance means that each level of an attribute should occur an equal number of times in the survey. Orthogonality essentially means that estimable effects should not be correlated. Minimal overlap stipulates that attribute levels should be repeated within choice sets as little as possible. Utility balance attempts to balance the utility of the alternatives within a choice set. It is not generally possible to simultaneously uphold all four of these design principles. One popular quantitative measure of design efficiency is D-error = $|\ |^{1/k}$, where is the covariance matrix of the maximum likelihood estimator in the conditional logit model and k is the number of parameters in the model. By minimizing D-error, the researcher can approximately satisfy the four design principles.

Clearly, utility balance can only be achieved when the researcher has some a priori information about the parameters to be estimated. Huber and Zwerina [13] show that even when the parameter estimates are incorrect there are efficiency gains from using them in the survey design. The SAS choiceff macro directly minimizes D-error to generate efficient choice designs for the conditional logit model, allowing the researcher to use a priori estimates on model parameters. No research exists on design efficiency for more complicated models, like the one proposed in this paper.

repeated in the survey that there is no di¤erence in the probability of cleanup for a policy with immediate bene…ts versus one with delayed bene…ts.

4.2 Italian Money Data

I also have money choice data from Alberini and Chiabai's 2004 survey of 776 Italian residents [2]. In this survey, respondents choose between a hypothetical immediate lump-sum payment and a hypothetical stream of constant payments over 10 years. The lump-sum payment option is always e10,000 received now. The stream of constant payments option is varied with annual payments of e1150, e1500, or e1650. The respondents also have a third option of being indi¤erent between the lump-sum and the annuity. In this analysis, I throw out the observations for which the respondent is indi¤erent (64 observations lost) since an individual cannot be indi¤erent between alternatives in the random utility model. This leaves 712 observations.

4.3 State Lottery Lump Sum vs. Annuity Choice Data

In addition to the two stated-preference data sources already presba to the lump sum option, one can calculate the implicit interest rate of the annuity. The implicit interest rate is the rate that equates the present value of the annuity stream to the lump sum option. An individual prefers the lump sum payment over the annuity payments if the lump sum value exceeds their own internal present value of the annuity. Equivalently, an individual prefers the lump sum payment over the annuity payments if their internal (exponential) discount rate is higher than the implicit interest rate offered in the annuity. The less patient the individual, the more likely they will be to take the lump sum option. By observing the choices that winners make between the two options at multiple implicit interest rates, I am able to identify the average discount rate for lottery winners.

All three of these state lotteries advertise the dollar amount of the annuity option. Colorado and Florida allow winners to choose whether they want the lump sum or the annuity option after winning when claiming the prize. However, Texas requires winners to select their payment option when purchasing the ticket. Texas provides information to lottery players about the estimated lump sum payment for a given drawing. Therefore, I use the actual lump sum and annuity options offered to winners for Colorado and Florida but rely on the advertised lump sum and annuity options available to Texas lottery winners at the time of ticket purchase. Currently,

Table 6: Summary of State Lotteries

Lottery	Date Range	N	Lump Sum / Annuity	Implicit Annuity Interest Rate	% Choosing Lump Sum
Colorado Lotto 40%	08/20/1994–10/25/2003	177	40%	9.98%	60.45%
Colorado Lotto 50%	11/12/2003–1/05/2008	37	50%	6.97%	86.49%
Texas Lotto	10/27/2001–12/08/2007	74	54.7% to 64%	5.89% to 4.16%	82.43%
Florida Lotto 30-yr	11/28/1998–12/22/2007	343	42.5% to 70.3%	7.45% to 2.64%	91.80%
Florida Lotto 20-yr	10/24/1998–11/14/1998	5	64.5% to 64.7%	5.2% to 5.15%	60.00%
Total		636			81.43%

Note: The implicit annual interest rate is the interest rate that equates the present value of the sum of annuity payments to the lump sum option.

5 Estimation Results

Since the parameters enter choice utility in a nonlinear fashion it is necessary to write

Table 9: Minnesota River Basin Maximum Likelihood Results: Speci…cation I

Variable	I.a.	I.b.	I.c.	I.d.
Basin Improvement	0.02476 (0.00261)	0.02496 (0.00259)	0.02462 (0.00260)	0.02493 (0.00267)
Cost	-0.00298 (0.00022)	- 0.00302 (0.00022)	-0.00303 (0.00022)	-0.00304 (0.00023)
Harvey ( ) Parameter	0.38926 (0.03762)			
HM (!) Parameter		0.14756 (0.02156)		
Exponential ( ) Parameter			0.90740 (0.00905)	
Quasi-Hyperbolic ( ) Parameter				0.92871 (0.12708)

Table 10: Minnesota River Basin Maximum Likelihood Results: Speci…cation II

Variable	II.a.	II.b.	II.c.	II.d.
Basin Improvement	0.03216 (0.01116)	0.03532 (0.01134)	0.03652 (0.01176)	0.03579 (0.01165)
Cost	-0.00283 (0.00078)	- 0.00301 (0.00079)	-0.00310 (0.00079)	-0.00305 (0.00079)
Improvement X Age	-0.00023 (0.00014)	-0.00024 (0.00014)	-0.00024 (0.00015)	-0.00024 (0.00016)
Improvement X Income/10000	0.00276 (0.00081)	0.00259 (0.00079)	0.00246 (0.00078)	0.00258 (0.00083)
Improvement X Male	-0.00247 (0.00590)	-0.00340 (0.00573)	-0.00368 (0.00563)	-0.00332 (0.00581)
Improvement X Resident	-0.00776 (0.00466)	-0.00674 (0.00455)	-0.00602 (0.00449)	-0.00659 (0.00465)
Improvement X Education	-0.00097 (0.00161)	-0.00148 (0.00158)	-0.00172 (0.00156)	-0.00152 (0.00158)
Cost X Income/10000	-0.00019 (0.00007)	-0.00019 (0.00007)	-0.00019 (0.00007))	-0.00019 (0.00007)
Cost X Male	0.00021 (0.00053)	0.00027 (0.00053)	0.00029 (0.00053)	0.00027 (0.00053)
Cost X Resident	0.00060 (0.00043)	0.00055 (0.00043)	0.00051 (0.00043)	0.00055 (0.00044)
Cost X Education/100	0.00781 (0.01531)	0.01096 (0.01537)	0.01275 (0.01538)	0.01140 (0.01540)
Harvey ( ) Parameter	0.39486 (0.04111)			
HM (!) Parameter		0.15102 (0.02525)		
Exponential ( ) Parameter			0.90658 (0.01123)	
Quasi-Hyperbolic ( ) Parameter				0.88830 (0.11928)
Quasi-Hyperbolic ( ) Parameter				0.91197 (0.01330)

$(Y_{it} - c_{ijt}) + q_{ijt}x_{it} + c_{ijt}x_{it}] + {}_{ij}$; where x_{it} is a vector of personal characteristics for individual i at time t. Personal characteristics that could potentially influence utility include age, income, sex, education level, and whether the respondent resides within the Minnesota River Basin. Attempts to estimate the model including the variable "Cost X Age" fail to converge. Therefore, I drop "Cost X Age" from the model and estimate Specification II with the remaining variables. Table 10 reports results for this interactions specification.

Results for the discounting parameters in Specifications II.a.-d. are similar to results from Specifications I.a.-d. Again, the exponential discounting model fits the data better than the two single-parameter hyperbolic models. Viewing II.c. as a restricted model of II.d. I can again perform a likelihood ratio test. The test statistic is equal to .792 so I fail to reject the null hypothesis that $= 1$: There is no evidence in this interactions model in support of quasi-hyperbolic discounting.

In specification III, I assume that discounting parameters are random coefficients. Specifically, I assume that discounting parameters vary over people but are constant over choice situations for each person. In III.a., I assume hyperbolic discounting with the single parameter, ${}_i$, being distributed normally with mean $\bar{}$ and variance z^2. In III.b., I assume that the single parameter for HM hyperbolic discounting, $!_i$, is distributed normally with mean $\bar{!}$ and variance $z_!^2$. III.c. assumes exponential discounting with a discount factor, ${}_i$, that is distributed normally with mean $\bar{}$ and variance z^2. Finally, III.d. assumes quasi-hyperbolic discounting with a constant factor and a ${}_{Hi}$ factor that is distributed normally with mean $\overline{{}_H}$ and variance z^2_H. I derive the Simulated Log Likelihood equation in appendix A. Attempts to treat both the factor and the ${}_H$ factors as random fail to converge.

Table 11 gives results for the random coefficients specifications. The maximized value of the simulated log likelihood equation is greater in the exponential specification (III.c.) than in either of the single parameter hyperbolic specifications (III.a. and

Table 11: Minnesota River Basin Simulated Maximum Likelihood Results: Speci...ca-tion III

Variable	III.a.	III.b.	III.c.	III.d.
Basin Improvement	0.52481	0.41877	0.45925	0.44873
	(0.08040)	(0.05772)	(0.05906)	(0.09564)
Cost	-0.07194	- 0.05439	-0.06271	-0.06170
	(0.00813)	(0.00638)	(0.00680)	(0.00993)
Harvey ( ) Parameter Mean	0.50596			
	(0.09250)			
Harvey ( ) Parameter S.D.	0.45465			
	(0.09364)			
HM (!) Parameter Mean		0.32920		
		(0.04742)		
HM (!) Parameter S.D.		0.14180		
		(0.01616)		
Exponential ( ) Mean			0.87976	
			(0.02154)	
Exponential ( ) S.D.			0.10377	
			(0.02616)	
Quasi-Hyperbolic ( ) Parameter				1.02834
				(0.20711)
Quasi-Hyperbolic ( ) Mean				0.87761
				(0.02727)
Quasi-Hyperbolic ( ) S.D.				0.10558
				(0.02941)
Simulated Log L	-1222.1843	-1229.4897	-1218.8206	-1218.8106

Note: Asymptotic Standard Errors are given in parenthesis.
* signi...cant at 10%, **signi...cant at 5%, *** signi...cant at 1%

anchored to the present in all choices.

I apply equations 17, 18 and 22 to the Italian money data set. Assuming exponential discounting, maximum likelihood estimation gives b

Table 12: Results for State Lottery Data: N=636

Discounting Model	Parameter Estimate	Log L
Harvey Hyperbolic	0.375 (0.001623)	-865.708
HM Hyperbolic	0.134 (0.000945)	-837.749
Exponential	0.927	-822.686

data fail to reject the null hypothesis of standard exponential discounting. Estimates of the constant exponential discount rates range from approximately eight to eleven percent throughout the three data sets.

I find evidence that individuals do behave rationally when making intertemporal decisions. They are dynamically consistent in their choices and do not appear to be present-biased. The range of discount rates estimated here falls below the discount rates commonly found in the experimental literature but is consistent with interest rates that we see in capital markets, as we would expect from theory. From a policy perspective, these results have implications for a variety of contexts including personal savings decisions, participation in preventative health programs, the formation of human capital, and environmental sustainability.

Because of the nature of the original data sets employed in this paper, confounding factors that are commonly part of experimental studies are minimized. Specifically, the data sets minimize perceived uncertainty in the receipt of future rewards, perceived future transaction costs, and the correlation between the length of delay before a future outcome and the length of the interval between two outcomes. I propose that much of the prior evidence for hyperbolic discounting may be questionable when these confounding factors are considered.

A Random Coefficients Simulated Log Likelihood Equation

Here, I develop the simulated log likelihood equation for the random coefficients specification. For clarity, I present the exponential discounting case. All other discounting models are easily derived with a few substitutions. This section loosely follows the exposition of Train. [27]

Recall the probability of a single choice for the non-stochastic discounting parameters case, $P_{ij} = F\left(\frac{\sum_{t=0}^{T_j} {}^t v_{ijt} - \sum_{t=0}^{T_k} {}^t v_{ikt}}{\sqrt{\sum_{t=0}^{T_k} {}^{2t} + \sum_{t=0}^{T_j} {}^{2t}}}\right):$ In the case of random discounting parameters, I focus on the sequence of choices by individual i: Denote the choice situation as s and a sequence of alternatives as $j = \{j_1; :::; j_S\}$ Then, conditional on , the probability that individual i makes a sequence of choices is the product over all s of the single choice probabilities. I have

$$_{ij}(\;) = \prod_{s=1}^{S} F\left(\frac{\sum_{t=0}^{T_{j;s}} {}_i^t v_{ijts} - \sum_{t=0}^{T_{k;s}} {}_i^t v_{ikts}}{\sqrt{\sum_{t=0}^{T_{k;s}} {}_i^{2t} + \sum_{t=0}^{T_{j;s}} {}_i^{2t}}}\right): \tag{23}$$

Since the are random, I integrate out over all values of to get the unconditional choice probability

$$P_{ij} = \int {}_{ij}(\;) f(\;) d\; : \tag{24}$$

I draw R values of from f( ) and denote them $_r$: The simulated choice probability is $\check{P}_{ij} = \frac{1}{R} \sum_{r=1}^{R} {}_{ij}(\;_r)$: In this application, I set R = 200: Finally, I insert these simulated choice probabilities into the log-likelihood function to get the simulated log likelihood (SLL)

$$SLL = \sum_i \sum_j y_{ij} \ln \check{P}_{ij}; \tag{25}$$

where $y_{ij} = 1$ if i chose sequence j and zero otherwise.

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