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Does Evolution Solve the Hold-up Problem?

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1 Introduction

[illegible]

2 Investment and Bargaining

[illegible] $D_A \mapsto \{V\ I() - x; V\ I()\}$ [illegible] B [illegible] A [illegible] A [illegible] D [illegible] B [illegible] A [illegible] $I; y\ x()$ [illegible] A [illegible] y, $D \to D$ [illegible]

[illegible] A [illegible] $I \mapsto I^*$ [illegible] I

3 Evolution

[illegible] *population* [illegible] N [illegible] $t \in \{1, \ldots\}$ [illegible] A [illegible] B [illegible] *beliefs* [illegible] A [illegible] B [illegible] B [illegible] A [illegible] I [illegible] B [illegible] x [illegible]

Assumption 1 *(i) The pie division is small:* $V \; I > -$. *(ii) The population is large:* $V \; I^{*}$

adaptation

random mutation.

Θ

basin of attraction

μ *B* μ( ) μ′

μ′ μ μ′ *single mutation neighborhood* μ μ′ ∈ *M* μ( ) μ′ μ

X *mutation connected set*

μ_1 $\mu_n \subset X$ $\mu_2, \ldots, \mu_{n-1}$

k ; ...; *n*; *M* μ( ) ∩ *BX*μ *n*

A A $I^*; V(I^*) - -$;

I^*

B

$$x^L := \max\{x \in D_B(I^*) | (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I - \}:$$

x^L B A

$$x^M := \max\{x \in D_B(I^*) | V(I^*) - x - I^* > V(I) - - - I\}:$$

$x^M - \leq x^L \leq x^M$:

$$\begin{aligned} V^* - x^M - (N-1)/N - I^* &:= (V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\ &\quad - (N-1)/N \\ &> V(I^*) - x^M - I^* - - - x^M)/N \\ &\geq V(I^*) - x^M - I^* \\ &\geq V(I) - - - I; \end{aligned}$$

$(N-1)/N$ x^L B A

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ $:= (I^*; V(I^*) - x; x)$, where $x \leq x^L$.*

resistance *cost*

stochastic potential

Theorem 1 *An equilibrium* μ *is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

$r\ x_{()} > r\ x_{()}$ $x > x^{L}$ $()$

$x^{M} > x^{NBS}$

$()$

$r\ x^{L}_{()} \geq r\ x^{L} - -_{()};$

$$- \longrightarrow \ - \longrightarrow ::: \longrightarrow x^{L} - - \longrightarrow x^{L};$$

$r\ x^{L}_{()} \geq r\ x^{L} - -_{()}$

$$x^{L} \longrightarrow - \longrightarrow \ - \longrightarrow ::: \longrightarrow x^{L} - -:$$

Appendix: Proofs

Lemma 1 *Let $z_1 < z_2 ::: < z$ be demands in $D\ I()$ for some $I \in \Psi$. Assume that the set of demands following I for agents in the relevant population is* $\{z_l\}_l$

[illegible] $I;y$ [illegible] A [illegible] $I;y$ [illegible] μ' [illegible] $\square$

Lemma 4 *Let* μ' *(*$\mu'(\cdot) = \{ I';y';x' \}$*) be an equilibrium. If* $I \neq I'$ *and* $y - I \geq y' - I'$*, then the population can get from* μ' *to an equilibrium* μ *with* $\mu(\cdot) = \{ I;y;x \}$ *through a sequence of single mutation transitions.*

[illegible] μ' [illegible] B [illegible] A [illegible] I [illegible] y [illegible] j [illegible] B x I [illegible] x [illegible] $V(I) - y$ [illegible] A [illegible] $I;y$ [illegible] A [illegible] x I [illegible] x [illegible] j [illegible] B [illegible] $y - I > y' - I'$ [illegible] $I;y$ [illegible] μ [illegible] $\mu(\cdot) = \{ I;x;y \}$ [illegible] $y - I = y' - I'$ [illegible] μ_1 [illegible] $\mu_1(\cdot) = \{ I;y;x ; I';y';x' \}$ [illegible] μ [illegible] $\mu(\cdot) = \{ I;y;x \}$ $\square$

[illegible] μ [illegible] $\mu(\cdot) = \{ I;V(I) - x;x \}$ [illegible] $I \neq I^*$ [illegible] $V(I^*) - - - I^* > V(I) - x - I;$ [illegible] μ^L [illegible] $\mu^L(\cdot) = \{ I^*;V^* - -;- \}$ [illegible] $I = I^*$ [illegible] $x > x^L$ [illegible] $V(I) - - - I \geq V(I) - x - I$ [illegible] μ [illegible] $\mu(\cdot) = \{ I;V(I) - -;- \}$ [illegible] μ^L [illegible] $\mu^L(\cdot) = \{ I^*;V^* - -;- \}$ [illegible] $I = I^*$ $x > x^L$ [illegible] $V(I) - - - I < V(I) - x - I$

[(st3 1 Tn)411

$I; V \; I() - -()$

$\{I^*; y; x()\}$ $x \leq x^L$

μ_1

$V \; I^* () - x^L () \; N - I () = N - I^* \leq V \; I^* () - x () \; N - I () = N - I^*$

$A \; x() \; \mu i$

A ... $(I; V - -; -)$ ... B ...

$x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome* $(I^*; y; x)$ *with* $x \leq x^L$ *to an equilibrium with outcome* $(I; V - -; -)$ *is* $r(x) = \min\{r | r > N(1 - \frac{\hat{V}-\delta-\hat{I}+I^*}{V^*-})\}$.

... r ... B ...

$$\frac{N-r}{N}(V^* - x) - I^* < V - I - -; \qquad (\;)$$

... r ... $\square$

Lemma 6

(i) If µ *is an equilibrium with outcome* $(I^*; y; x)$ *and* $x < \min\{x^M; x^{NBS}\}$;

the easiest transition away from ‰ µ ...

$x > x^L$

$x = x^L = x^M$ $x^M < x^{NBS}$;

$r(x) \geq r(x)$ $V^* - x^M - - \leq V - - - I$ □

$x = x^M > x^{NBS}$

$I; V - -; -)$ $(I^*; y$ $-; x - -)$

$(I^*; y; x)$ $y = V^* - x)$ $(I^*; y'; x')$

Lemma 7 *From an outcome $(I^*; y; x)$ the easiest transition in which investment is at all times e–cient, but which ends with difierent demands, is to an outcome $(I^*; y'; x')$ where $x = x - -; x$ $-; -;$ or $V^* - -$.*

□

$-;$

$-$

Lemma 8

(i) Moving from x to $x - -$ takes $N(1 - \frac{-\delta}{})$ mutations to pop A.

(ii) Moving from x to x $-$ takes $N(1 - \frac{V^* - -\delta}{V^* -})$ mutations to pop B.

(iii) Moving from x to $-$ takes $N(\frac{V^* -}{V^* - \delta})$ mutations to pop B.

(iv) Moving from x to $V^* - -$ takes $N(\frac{}{V^* - \delta})$ mutations to pop A.

N □

Lemma 9

(i) If $- < x < V^* - -$, then moving from x to $x - -$ takes fewer mutations than moving from x to $-$, and moving from x to x $-$ takes fewer mutations than moving from x to $V^* - -$.

(ii) If x $_{\vdash}$ *– then moving from x to*

[illegible] $x^L > x^{NBS}$ [illegible] Θ^1 [illegible] Γ^1 [illegible] $x < x^{NBS}$ [illegible] $\mu, \mu'_{+\delta}$ [illegible] $x^{NBS} < x \le x^L$ [illegible] $\mu, \mu'_{-\delta}$ [illegible] $r(x) < r(x-)$ [illegible] $x < x^{NBS}$ [illegible] $r(x) > r(x-)$ [illegible] $x > x^{NBS}$. $\square$

[illegible] A [illegible] B [illegible] B [illegible] l [illegible] A [illegible] y [illegible]

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))$, is a subset of the unique locally stable set.*

[illegible] $\exists \mu^H$ [illegible] $\mu^H \in T(\mu)$ [illegible] μ [illegible] ‰$(\mu^H) = \{(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))\}$ [illegible] μ [illegible] ‰$(\mu) = \{(l, V(l) - x, x)\}$ [illegible] A [illegible] $x^{\max}(l')$ [illegible] $l' \ne l$ [illegible] $x^{\max}(l)$ [illegible] l [illegible] B [illegible] $x^{\max}(l)$ [illegible] B [illegible] A [illegible] l^H, [illegible]

$\square$

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - l^H - x^{\max}(l^H)$ in every equilibrium.*

[illegible] l^H [illegible] A [illegible] l^H. $\square$

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V(l) - l - x \ge V^H - l^H - x^{\max}(l^H)$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and* ‰$(\mu) = (l, V(l) - x, x)$.

6 References

[illegible] *Journal of Economic Theory* [illegible]

[illegible] *Economic Journal* [illegible]

[illegible] *Review of Economic Studies* [illegible]

[illegible] *Games and Economic Behavior* [illegible]

[illegible] *Quarterly Journal of Economics* [illegible]

[illegible]

[illegible] *American Economic Review* [illegible]

[illegible] *Theoretical Population Biology* [illegible]

[illegible] *Games and Economic Behavior* [illegible]

[illegible] *Journal of Political Economy* [illegible]

[illegible] *Econometrica* [illegible]

[illegible] *Journal of Law, Economics and Organization* [illegible]

Markets and Hierarchies: Analysis and Antitrust Implications,

Econometrica

Journal of Economic Theory

Review of Economic Studies