

$\langle \cdot, \cdot \rangle$ is the inner product in $L^2(\mathbb{R}^d)$. For $\phi \in L^2(\mathbb{R}^d)$, we define the adjoint operator \mathcal{L}^* by $\langle \mathcal{L}\phi, \psi \rangle = \langle \phi, \mathcal{L}^*\psi \rangle$. The adjoint operator \mathcal{L}^* is given by $\mathcal{L}^*\psi = -\operatorname{div}(\mathbf{D}\psi) + \mathbf{D}\psi \cdot \mathbf{c}$, where $\mathbf{D}\psi = (\partial_{x_i}\psi)_{i=1,\dots,d}$ is the gradient of ψ and $\mathbf{c} = (c_i)_{i=1,\dots,d}$ is the drift vector field. The adjoint operator \mathcal{L}^* is self-adjoint if and only if $\mathbf{c} = -\operatorname{div}(\mathbf{D}\mathbf{c})$. The adjoint operator \mathcal{L}^* is also symmetric if and only if $\mathbf{c} = -\operatorname{div}(\mathbf{D}\mathbf{c})$ and $\mathbf{c} \cdot \mathbf{c} = \operatorname{div}(\mathbf{D}\mathbf{c})$.

As $\gamma \rightarrow \infty$, $\theta \rightarrow 0$, and $\theta \rightarrow 0$.

$$(1.1) \quad \theta \rightarrow 0, \quad \theta \rightarrow 0,$$

As $\gamma \rightarrow \infty$, $\theta \rightarrow 0$, and $\theta \rightarrow 0$.

As $\gamma \rightarrow \infty$, $\theta \rightarrow 0$, and $\theta \rightarrow 0$.

$$\frac{\theta}{\gamma} \rightarrow 0, \quad \theta \rightarrow 0, \quad \theta \rightarrow 0,$$

As $\gamma \rightarrow \infty$, $\theta \rightarrow 0$, and $\theta \rightarrow 0$.

$$(1.2) \quad \theta \rightarrow 0, \quad \theta \rightarrow 0, \quad \theta \rightarrow 0,$$

As $\gamma \rightarrow \infty$, $\theta \rightarrow 0$, and $\theta \rightarrow 0$.

$$\theta \rightarrow 0,$$

$(\cdot, \cdot)_{\lambda} = (0, \cdot)$, $\lambda = 0$, $\beta = \alpha$, $\lambda = \beta$, α , β , (\cdot, \cdot) , β , (\cdot, \cdot) , β , (\cdot, \cdot) .

... ζ ... ζ ...

$$\alpha \rightarrow \alpha \frac{1}{\alpha},$$

$$(\alpha + \alpha) + \int_{-\pi}^{\pi} (\dots)'(\dots) (\dots) \frac{1}{\alpha}$$

... Ω ... 0 ... Ω ... Ω ... Ω ... 0 ... 0 ... 0 ... Ω

$$\begin{aligned}
& \Psi, \Psi, \Psi \\
& \dots * \dots fi \\
& (\dots) * \dots \frac{f(\dots)}{f} \dots + \alpha(\dots) \dots + \frac{f'(\dots)}{\alpha(\dots)} \frac{\pi}{-\pi} (\dots) (\dots) \\
& \dots fi \\
& \zeta^*(\dots) = \frac{f'(\dots)}{f} \dots \zeta^*(\dots)
\end{aligned}$$

Downloaded 05/23/14 to 130.49.198.5. Redistribution subject to SIAM license or copyright; see http://www.siam.org/journals/ojsa.php

... $\delta \in \mathbb{R}$

$$(\alpha + \beta) \psi(\zeta) + \frac{\pi}{-\pi} \psi'(\zeta) \psi(\zeta) = \frac{\psi''(\zeta)}{\alpha}$$

... , ...

$$\frac{\Psi}{\alpha}, \zeta^* \psi(\zeta) = \delta \frac{\zeta}{\alpha}, \zeta^* \psi(\zeta) - \frac{\psi''(\zeta)}{\alpha} = 0, \zeta^* \psi(\zeta) = \frac{\psi''(\zeta)}{\alpha}$$

$$\psi''(\zeta) = \frac{\pi}{-\pi} \psi'(\zeta) \psi(\zeta) - \psi''(\zeta) \psi(\zeta)$$

$$(\dots) = \frac{\psi''(\zeta)}{\alpha} \psi(\zeta)$$

... $\psi(\zeta)$...

$$(\alpha + \beta) \psi''(\zeta) = \frac{\pi}{-\pi} \psi(\zeta) \psi''(\zeta)$$

$$\psi''(\zeta) = \frac{\pi}{-\pi} \psi'(\zeta) \psi''(\zeta) - \psi''(\zeta) \psi(\zeta) = \frac{\pi}{-\pi} \psi(\zeta) \psi''(\zeta)$$

$$= \frac{\pi}{-\pi} \psi(\zeta) \psi''(\zeta)$$

$$= \frac{\pi}{\alpha} \frac{\pi}{-\pi} \psi(\zeta) \psi''(\zeta) = 0$$

... (\dots) ... α, β ... (\dots)

$$(\dots) = 0, \quad \pm \psi(\zeta) = \pm \alpha(\beta - \alpha) \frac{\psi'(\zeta)}{\psi''(\zeta)}$$

... (\dots) ...

$$\psi(\zeta) = (\alpha + \beta) \psi(\zeta) + \psi(\mu)$$

$$\psi(\zeta) = (\alpha + \beta) \frac{\psi'(\zeta)}{\alpha} + \psi(\mu)$$

¹N ... (1.2), ... = ... , $\|\psi\|_{\mathcal{F}}^2 = |\langle \psi, \psi \rangle|$... $\pm = \pm \sqrt{\alpha(\beta - \alpha)}$. A ... 3.1, ...

$\pm(\cdot, \cdot) \approx (\xi)$, $\pm(\cdot, \cdot) \approx (\xi \pm \alpha)$, $\xi \approx \pm$

$\pm(\cdot, \cdot) \approx (\xi)$, $\pm(\cdot, \cdot) \approx (\xi \pm \alpha)$, $\xi \approx \pm$

2.3. Perturbed amplitude equations at the drift bifurcation.

$(\cdot, 0)$ $\mu \approx \beta \alpha = 0$

$(\cdot, \cdot) \sim \bar{\mu}(\mu)$ $\mu \rightarrow 0$,

$$\bar{\mu}(\mu) \approx \bar{\mu} + \mu \bar{\mu}' + \frac{\mu^2}{2} \bar{\mu}'' + \dots$$

$(\cdot, 0)$, $\Delta(\cdot) \sim \bar{\mu}$

$$(\cdot, \cdot) \quad \bar{\mu}_\mu(\cdot, \cdot) \approx (\cdot, \cdot) \beta(\cdot, \cdot) + \frac{\pi}{2} (\cdot, \cdot) ((\cdot, \cdot))$$

$$(\cdot, \cdot) \quad \bar{\mu}_\mu(\cdot, \cdot) \approx \alpha \lambda((\cdot, \cdot) - (\cdot, \cdot)) + \mu^{\frac{3}{2}} (\cdot, \cdot)$$

(\cdot, \cdot) , $\bar{\mu}_\mu(\cdot, \mu^{-1/2}(\cdot, \mu))_{t \geq \dots}$

$$(\cdot, \cdot) \quad (\cdot, \cdot) \approx (\Delta(\cdot)) + \mu (x \Delta(\cdot), \cdot) + \mu^{\frac{3}{2}} (\Delta(\cdot), \cdot) + \dots$$

$$(\cdot, \cdot) \quad (\cdot, \cdot) \approx (\Delta(\cdot)) - \bar{\mu}(\cdot)$$

... .. (. .) . . .

(. .) .. ((Δ(),) . (Δ(),) ≤ $\frac{\mu}{\alpha}$ '''(Δ()) + $\mu^{\frac{1}{2}}$

... .. $(\mu^{\frac{3}{2}})$..

(. .) .. ((Δ(),) ≤ ((Δ(),)) + $\frac{f'(\Delta(\cdot))}{\alpha}$] ,

.. ((Δ(),) ≤ $\frac{f'(\Delta(\cdot))}{\alpha}$ + α((Δ(),) . (Δ(),))

(. .) .. + .. (,] ,

... .. fi ..

... () , ≤ (+ α) () + $\int_{-\pi}^{\pi} (\cdot)' f' ((\cdot)) (\cdot)' ..$

... .. (. .) .. (. .) ,

.. (..] ≤ $\frac{f'(\cdot)}{\alpha}$ + $\frac{f'(\cdot)}{\alpha}$ $\frac{\mu}{\alpha}$ ''' [..] .. (,])

... .. < α f' () f' ,

... .. + ..] α, (] (Ψ) Ω .. Ω 0 . 0 0 , .. Ω . 0

$$\begin{aligned}
 \Delta \mu & \sim \frac{1}{\sqrt{N}} \sum_{i=1}^N \left(\xi_i - \mu \right) \\
 \Delta \mu & \sim \frac{1}{\sqrt{N}} \sum_{i=1}^N \left(\xi_i - \mu \right) \\
 \Delta \mu & \sim \frac{1}{\sqrt{N}} \sum_{i=1}^N \left(\xi_i - \mu \right)
 \end{aligned}$$

Downloaded 05/23/14 to 130.49.198.5. Redistribution subject to SIAM license or copyright; see http://www.siam.org/journals/ojsa.php

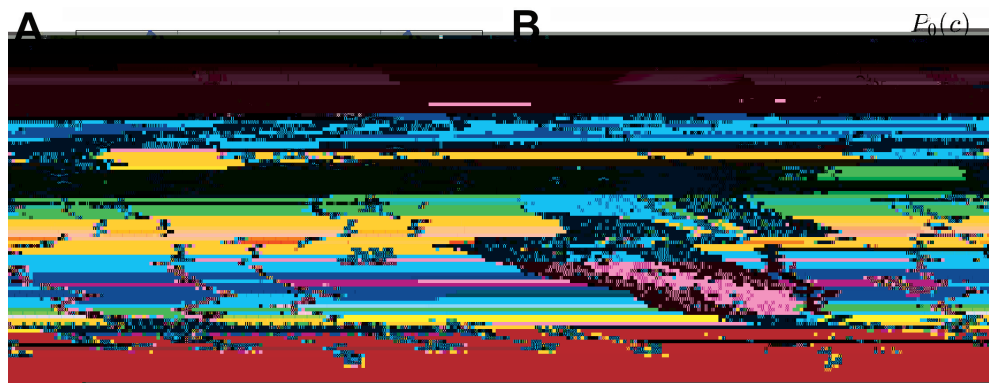


Figure 2. \mathcal{G}_1 is a \mathbb{Z}^d -invariant random walk on \mathbb{Z}^d with transition probabilities $p_{ij} = \frac{1}{2d} \mathbb{1}_{\|i-j\|_1=1}$. Let $\alpha \in (0, 1)$ and $c \in (0, 1)$. Let $\mathcal{C}(c) = \{x \in \mathbb{Z}^d : \mathcal{G}_1 \text{ visits } x \text{ infinitely often}\}$. (A) $\mathcal{C}(c)$ for $c = 0.5$. (B) $\mathcal{C}(c)$ for $c = 0.9$. The labels \mathcal{A} and \mathcal{B} are placed above the corresponding panels. The label $E_0(c)$ is placed at the top right of the figure.

$$\begin{aligned}
 & \dots \\
 & \dots (\cdot) = \int_{-\pi}^{\pi} (\cdot)'(\cdot + \dots) \dots \\
 & \dots (\cdot), \dots (\cdot), (\cdot, \cdot). \\
 & \dots \text{fi} \dots (\cdot), \dots \\
 & \dots \text{fi} \dots \theta \quad \xi = \pi. \\
 & \dots \\
 & (\pi) = \dots
 \end{aligned}$$

π_0
 \mathbb{R}^2
 $(\xi, q(\xi))$
 γ_{\pm}
 (ξ)

$$\gamma_+ + \gamma_-$$

Downloaded 05/23/14 to 130.49.198.5. Redistribution subject to SIAM license or copyright; see <http://www.siam.org/journals/ojsa.php>

- [7] D. Blömker, M. Hairer, and G. A. Pavliotis, *Stochastic approximation with time-varying gain*, *SIAM J. Numer. Anal.*, 20 (2007), pp. 1721–1744.
- [8] D. Blömker, S. Maier-Paape, and G. Schneider, *Stochastic approximation with time-varying gain*, *SIAM J. Numer. Anal.*, 1 (2001), pp. 527–541.
- [9] M. Bode,

- [33] C. W. Gardiner, *Stochastic Processes: Classical and Quantum*, Wiley, 2009.

- [59] M. Tsodyks, K. Pawelzik, and H. Markram, *Nature*, 10 (1998), . 821–835.
- [60] R. Veltz and O. Faugeras, *Journal of the Royal Society B*, 9 (2010), . 954–998.
- [61] X.-J. Wang, *Nature*, 90 (2010), . 1195–1268.
- [62] H. R. Wilson and J. D. Cowan, *Biological Cybernetics*, 13 (1973), . 55–80.
- [63] W. Xu, X. Huang, K. Takagaki, and J.-Y. Wu, *Chaos*, 55 (2007), . 119–129.
- [64] K. Yoon, M. A. Buice, C. Barry, R. Hayman, N. Burgess, and I. R. Fiete, *Science*, 16 (2013), . 1077–1084.
- [65] L. C. York and M. C. W. van Rossum, *Chaos*, 27 (2009), . 607–620.