Numerical Analysis Preliminary Exam

January 17, 2012

Time: 180 Minutes

Do 4 and only 4 of the following 6 problems. Please indicate clearly which 4 you wish to have graded.

!!! No Calculators Allowed !!!

!!!Show all of your work !!!

NAME:	

For Grader Only

- 1. Nonlinear Equations Given scalar equation, f(x) = 0,
 - 1. Describe I) Newtons Method, II) Secant Method for approximating the solution.
 - 2. State su cient conditions for Newton and Secant to converge. If satis ed, at what rate will each converge?
 - 3. Sketch the proof of convergence for Newton's Method.
 - 4. Write Newton's Method as a xed point iteration. State su cient conditions for a general xed point iteration to converge.
 - 5. Apply the criterion for xed point iteration to Newton's Method and develop an alternate proof for Newton's Method.

Numerical quadrature:

2. Assume that a quadrature rule, when discretizing with des, possesses an error expansion of the form

$$I - I_n = \frac{C_1}{n} + \frac{C_2}{n^2} + \frac{C_3}{n^3} +$$

Assume also that we, for a certain value, of have numerically evaluated, I_{2n} and I_{3n} .

- a. Derive the best approximation that you can for the true **value** integral.
- b. The error in this approximation will be of the fo@(n-p) for a certain value Wafhat is this value for p?

Interpolation / Approximation:

3. The General Hermite interpolation proble amounts to finding a polynomia (x) of degree $1 + 2 + \dots + n - 1$ that satisfies

$$\begin{split} p^{(i)}(x_1) &= y_1{}^{(i)} \;, \quad i = 0, \; 1, \; ... \;\;, \; {}_1 \; \text{--} \;\; 1 \\ & : & : \\ p^{(i)}(x_n) &= y_n{}^{(i)} \;, \quad i = 0, \; 1, \; ... \;\;, \; {}_n \; \text{--} \;\; 1 \;\;, \end{split}$$

where the superscripts denotes derivatives, that is, we specify the first vatides at the point x_j , for j=1,2,...,n. Show that this problem has a unique solution wheneveratree distinct.

Hint: Set up the linear system for a small problem, recognize the pattern, and properties result.

4. Linear Algebra

Consider the n n, nonsingular matrix, A. The Frobenius norm of A is given by

$$kAk_F = \begin{pmatrix} X \\ i;j \end{pmatrix} j\partial_{i;j} j^2)^{1=2}$$

- 1. Construct the perturbation, @A, with smallest Frobenius norm such that A @A is singular. (Hint: use one of the primary decompositions of A.)
- 2. What is the Frobenius norm of this special @A?
- 3. Prove that it is the smallest such perturbation.
- 4. Extra Credit: Is it unique?

6. Partial Di erential Equations

Consider the steady-state, advection-di usion equation in one space dimension:

$$\mathscr{Q}_{x}(a(x)\mathscr{Q}_{x}u(x)) + b(x)\mathscr{Q}_{x}u = f$$
 2 [0:1]

with boundary conditions u(0) = u(1) = 0 and the assumption that a(x) is continuous and a(x) > 0 for $x \ge [0,1]$

1. Describe the $\,$ nite di erence (FD) method for approximating the solution using I) Centered Di erences, II) Upwind Di erences on the advection term. Let h