Applied Analysis Preliminary Exam

10.00am{1.00pm, August 20, 2013

Problem 1: Show that the non-linear integral equation:

$$v(x) = \cos^2(x) + \int_0 e^{-\ \ ^{2}(\ )} ds, \qquad x \in [0, \infty)$$

has a solution in $C^1([0,\infty),\mathbb{R})$.

Problem 2: Calculate the limit. **Justify** your answer.

$$\lim_{\to\infty} \sum_{=1} \sin\left(\pi\sqrt{\frac{k}{n}}\right)\frac{1}{\sqrt{kn}}.$$

Problem 3: Given a self-adjoint compact operator $A : \ell^2 \longrightarrow \ell^2$, we de ne, for $\lambda \in \mathbb{R}$,

$$E_\lambda = \overline{\text{Span}\{v \in \ell^2 \mid Av = \mu v \text{ for some } \mu \leq \lambda\}}$$

and let

$$E^\lambda = E_\lambda^\perp$$

denote the orthogonal complement of E_λ.

(a) Show that E^1 is nite dimensional and A maps it to itself.

(b) In general, for what kind of value λ can you guarantee that:
  (1) E_λ is nite dimensional
  (2) E_λ is in nite dimensional
  (3) E^λ is nite dimensional
  (4) E^λ is in nite dimensional

Problem 4: Let H be a Hilbert space with an orthonormal basis $(\varphi\ )_{=1}^\infty$. Suppose further that $(\lambda\ )_{=1}^\infty$ is a sequence of non-negative real numbers such that $\lambda\ \to \infty$ as $j \to \infty$. De ne for any nite positive integer n, the operator $A\ (t) \in \mathcal{B}(H)$ via

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