1. (34 points) Find the requested information. The problems are unrelated.

   (a) Evaluate $\int \frac{\tan^{-1}(x)}{x^2}\,dx$ (Hint: Start with IBP)

   (b) Find y as a function of x given that $\frac{dy}{dx} = 2x\sqrt{1-y^2}$ and $y(0) = 1$

   (c) Find the sum of the series $-\frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \cdots$

   (d) For what values of does the series $\sum_{n=0}^{\infty} \sin^2(\ )\cos^{2n}(\ )$ converge? Find the sum for those values of .

2. (16 points) Decide whether the following quantities are convergent or divergent. Explain your reasoning and name any test you use.

   (a) The sequence given by $a_n = \left(1 - \frac{\ln(3)}{n}\right)^n$, for $n = 1, 2, \ldots$

   (b) $\int_1^{\infty} \frac{1}{x^2}\sqrt{1 + \frac{3}{x^3}}\,dx$

3. (12 points) Consider the series $\sum_{k=1}^{\infty} b_k$. Suppose the n^{th} partial sum of the series is $s_n = 2 - \frac{2}{n+1}$.

   (a) What is s_3?

   (b) Find a simple formula for b_n.

   (c) What does $\{b_n\}$ converge to?

   (d) What is the sum of the series $\sum_{k=1}^{\infty} b_k$?

4. (25 points) Recall $\cosh(x) = \frac{e^x + e^{-x}}{2}$.

   (a) Find the MacLaurin series of the hyperbolic cosine function.

   (b) Find the interval of convergence for the power series from part (a).

   (c) Find $T_3(x)$, the Taylor polynomial of order 3, of the hyperbolic cosine centered at $a = 0$. Use the Taylor Remainder formula to find an upper bound for the absolute error if $T_3(x)$ is used to approximate $\cosh(1)$.

   (d) Use the MacLaurin series (not L'Hôpital!) to evaluate the following limit:

$$\lim_{x \to 0} \frac{\cosh(x) - 1 - \frac{x^2}{2}}{x^4}.$$

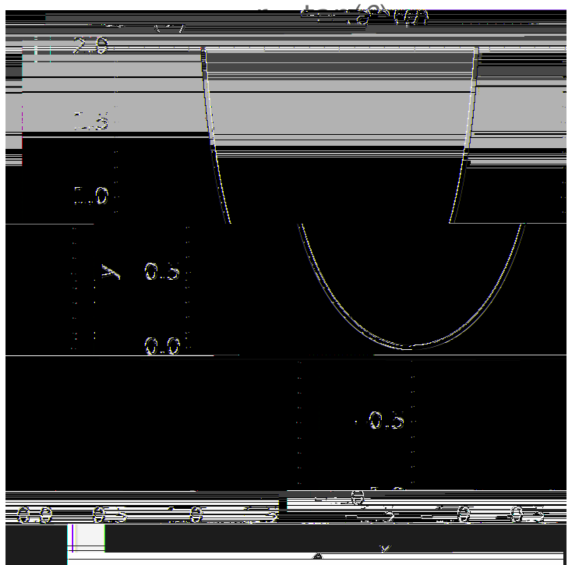
5. (18 points) Suppose $g(x)$ equals the power series $\sum_{n=2}^{\infty} \frac{(n+1)(x+b)^n}{c^{2n}}$, where b and c are constants, and the series has an interval of convergence of $-6 < x < 2$.

   (a) Find the center and radius of convergence of the series.

(b) Evaluate $\int g(x)\,dx$ as a power series.

(c) Given the interval of convergence, find possible values for b and c. Justify your answer using appropriate test(s).

6. (25 points) For this problem, let $r = \tan(\theta)$ for $-\pi/2 < \theta < \pi/2$. The polar graph (in the xy plane) is given below. Answer the following questions.



(a) Find an equation for the tangent line at $\theta = \pi/4$.